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BIG BAD RACISTS, SUBTLE PREJUDICE AND MINORITY VICTIMS: AN AGENT-BASED ANALYSIS OF THE DYNAMICS OF RACIAL INEQUALITY

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BIG BAD RACISTS, SUBTLE PREJUDICE AND MINORITY VICTIMS: AN AGENT-BASED OF THE DYNAMICS OF RACIAL INEQUALITY

How many racists does it take to maintain racial inequality? Historical evidence from the Jim Crow era suggests one needs a large number of racist advocates in various social arenas. More recent social scientific research, however, cites a significant decline in racist beliefs that have not been paralleled by declines in racial inequality. Hence, the strong hypothesized connection between racist attitudes and racial inequality was erroneous. Researchers have responded to this change by asserting that racial inequality does not require an abundance of racists, but only a system of biased (i.e., racialized) social institutions— or patterns of interaction—which can maintain racial inequality with a few/no racists. This solution, however, leads one to question how widespread systemic bias must be to maintain racial inequality—a derivative of the initial question. This paper examines these questions regarding how many racists—or biased institutional actors—it takes to create and maintain racial inequality using an agent based model of a Nash Bargaining game. The results reveal that one needs an enormous amount of discrimination to create and maintain racial inequality. However, when we allow non-discriminating agents (i.e., non-racists) to use the race of competitors in decision making via social learning, the need for discriminatory agents to maintain inequality is reduced to nil.

Introduction

A simple question that has puzzled scholars for over four decades is: *how many racists does it take to maintain racial inequality?* In the past, many believed that an overwhelming number of racists were needed to maintain inequality (Allport 1954; Becker 1971; Bonacich 1972; Brimmer 1997; DuBois 1899; Myrdal 1944; Park 1950; Shulman 1989). More recently, in response to research citing a decline in racist attitudes and persistent racial disparities in outcomes, many have questioned the notion that the maintenance of racial inequality requires an abundance of racists (Bonilla-Silva 1996; Ogbu and Davis 2003; Oliver and Shapiro 1995; Omi and Winant 1994; Sowell 1978; Wellman 1977). These scholars propose that racial inequality is perpetuated via a complex network of biased social institutions (i.e., structural race theory) and/or minority disinvestments (i.e., victim theory) which weakens the need for an abundance of racists to maintain racial inequality. While these alternative theories point to different agents of inequality to re-answer the original question, one is still left to ponder how complete structural discrimination—or blocked opportunity in the case of victim theory—must be to maintain racial inequality.

The aim of this paper is to examine this simple question regarding how many racists—or how much discrimination—it takes to create and maintain racial inequality. Drawing on Ridgeway's (1991) research on status construction theory, I use an agent-based model (i.e., mathematical simulation) of a Nash bargaining game to examine the emergence and maintenance of racial inequality among groups of artificial agents. The Nash bargaining game used here is representative of many general phenomena where actors jointly bid on their share of a finite social good (e.g., school resources, meeting time) and, consequently, develop attitudes about distributive justice (i.e., subsequent bids) that contribute to social inequality. In the simulation below, I insert a small number of discriminators—big bad racists—into the Nash bargaining game to determine the extent to which a small number of discriminators can lead to the emergence of racial inequality. The results indicate that it takes a large number of

discriminators to inspire inequality. However, when non-discriminating agents are allowed to invoke the race of competitors in bid determination via social learning—as is seen in Ridgeway’s empirical research (Ridgeway and Erickson 2000; Ridgeway et al. 1998; Ridgeway et al. 2009)—the need for discriminatory agents to maintain inequality is reduced to nil.

Although the method employed in this paper—agent based modeling—is not widely used, it is an increasingly popular methodological and theoretical tool for studying dynamic phenomena (Axelrod 1997; Axtell, Epstein and Young 2001; Centola, Willer and Macy 2005; Epstein 2006; Gilbert and Troitzsch 2005; Hanneman, Collins and Mordt 1995; Leik and Meeker 1995; Moss and Edmonds 2005). The method involves creating an artificial society where agents interact with one another according to certain rules and use bounded rationality in their decision-making (Epstein and Axtell 1996). One then analyzes how certain micro-level interactions—or rules on interaction—among agents contribute to the emergence and maintenance of a macro-level social phenomenon (i.e., inequality). Here, the method allows me to analyze how a few discriminating agents inspire a social cascade in the bids of non-discriminating agents which results in racial disparities in outcomes—a topic which is not amenable to survey research.

I begin the paper with a review of the literature on race and status construction theory. Then, I outline the rules of the Nash bargaining game and the methods I use to analyze the simulations. I present the bargaining game results in two steps using: 1) graphs depicting how average group inequality emerges under various conditions, and 2) a multivariate statistical analysis to highlight the simulation parameters that shape the emergence and maintenance of group inequality. I conclude the paper with a discussion of the implications of the simulation results and the promise of using agent-based models as an additional tool for social scientists studying race.

Background

Race remains a central social phenomenon in the United States—and the Western world. Although the election of a U.S. President of African descent suggests a decline in the social relevance of race, the significant disparities in unemployment, wages, wealth, health and mortality underscore the continued relevance of race in the lives of American citizens (Blau 2003; Conley 1999; Grodsky and Pager 2001; Hayward and Heron 1999; Henry 1997; McCall 2001; Oliver and Shapiro 1995; Stewart 2009; Williams 1999). The persistence of racial inequalities from Reconstruction through the Civil Rights movement, and up to the modern “post-racial era” presents a terrible quandary for scholars (Levy 1998; McDaniel 1998; Wilson 1978). Specifically, *how can a system of inequality persist in a country that values—and professes to embody—equality, justice and meritocratic achievement?* This quandary has led to a host of theories on the sources of racial inequality each with its own a policy prescription for improving, or overlooking, the dismal socioeconomic outcomes of many U.S. minority groups.

Theories of Racial Inequality

The recent literature on racial and ethnic inequality suggests that a host of factors are responsible for the persistent, significant racial disparities in socioeconomic and health-related outcomes.

Discrimination theory, for example, suggests there are a sizeable number of persons with a “taste for discrimination” that uniquely contribute to observed racial inequality (Becker 1971). This theory advances that discriminators embrace the skewed—though historically normative—belief that certain race groups are inherently inferior and regularly discriminate against members of these race groups in social interaction. From this viewpoint, discriminators are the driving force behind observed racial inequality.

Although discrimination theory has been championed by various scholars and social organizations across the past century (Allport 1954; Becker 1971; Bonacich 1972; Brimmer 1997; DuBois 1899; Myrdal 1944; Park 1950; Shulman 1989), academic support for the theory has waned for two

reasons.¹ First, survey research shows that the prevalence of discriminatory attitudes towards minority groups has significantly decreased since the civil rights era (Schumann et al. 1997). This is attributable to overtly racist attitudes becoming socially inappropriate, rather than a disappearance of the behavior of discrimination (Bendick, Brown, and Wall 1999; Fix, Galster and Struyk 1993; Heckman and Siegelman 1993; Pager 2001; Pager and Quillian 2005; Yinger 1993). The second reason for the decline in discrimination theoretic work is because recent race theory and research has championed that structural factors (e.g., segregation, educational environment) largely shape racial inequality (Bashi and McDaniel 1997; Bonilla-Silva 1996; Oliver and Shapiro 1995; Omi and Winant 1994; Wellman 1977).

Structural theory suggests that racial disparities in outcomes are a product of a widespread, institutionalized racial ideology that shapes interactions across the entire scope of American social contexts. Bonilla-Silva (1996) poignantly differentiated structural theory from discrimination theory noting “...racism, as defined by [discrimination theorists], does not provide adequate theoretical foundation for understanding racial phenomena” (474-5). He further contends that racism is an ideology that is part of a larger system (i.e., structure) of racial stratification. He writes that once this system is established “...race becomes an independent criterion for vertical hierarchy in society” (475). Race, from the structural perspective, is an integral piece of the social machinery that distributes private rewards and allocates an array of public privileges—it is more than an overtly racist attitude/behavior expressed by a faction of dominant group members.

A final theory of racial inequality is the minority victim theory. This theory is an outgrowth of the economic model of statistical discrimination which suggests that employers, instead of overtly practicing discrimination, use race as a proxy for the differential productivity of workers (Arrow 1973; Gould 1992, 1999). In response to this “statistical discrimination,” Loury (1995) suggests “...minorities may invest less in skills than majority workers because it is more difficult for them to achieve high-level

positions” (119). Thus, minorities, in this case the victims of statistical discrimination, become contributors to observed racial inequalities through disinvestment in skill.

A similar argument is made in Ogbu’s cultural theory of academic disengagement. Ogbu (1974, 1978, 1983; Fordham and Ogbu 1986; Ogbu and Davis 2003) asserts that lower black educational outcomes are a product of: 1) the U.S. social structure which limits opportunities for specific minority groups, and 2) an adaptive coping response to perceived/observed structural limitations whereby certain behaviors such as speaking Standard English (i.e., acting white) are stigmatized. In this case, minority students, the victims of low structural opportunity, become contributors to the level of racial inequality in educational achievement by disinvesting in “stigmatized” resources (i.e., training).

As one can see, the statistical discrimination and academic disengagement theories both point to minority disinvestments as playing a key role in racial inequality.² Minorities, in both theories, are argued to operationalize their victim status and significantly add to the level of racial inequality through disinvestment. Hence, the larger minority victim theory asserts that minorities are partly, equally or largely—depending on the author—to blame for racial inequality due to disinvestment in skills and resources (D’Souza 1995; McWhorter 2000; Ogbu 1974, 1978, 1983; Fordham and Ogbu 1986; Fryer and Torelli 2005; Ogbu and Davis 2003; Sowell 1978, 1983).

The three aforementioned theories of racial inequality—which are not exhaustive—all point to unique mechanisms which create/recreate the social privileges of dominant group members. In the case of victim theory, the source of the disparities is a rational response to low expected outcomes. Thus, victim theory asserts that minorities play a critical role (i.e., primary source) in determining racial inequality. Discrimination theory, in contrast to victim theory, locates the source of disparities in a host of racists who exert their extraordinary influence on the mass of minority actors. The final theory, structural, suggests that racial inequality is largely shaped by the social momentum of racism that is embedded in nearly every aspect of the social system. Racial inequality, from the latter view, can be

perpetuated in a world where dominant actors operate on a seemingly non-racist value set (Bonilla-Silva 1996, 2003; Wellman 1977).

Notably, the discrimination and structural theories of inequality are both predicated on a similar ideology of racism. Oliver and Shapiro (1995) define this ideology as "...a belief in the inherent inferiority of one race in relation to the other...[that] both justifies and dictates the actions and institutional decisions that adversely affect the target [minority] group." (34). In the case of discrimination theory, the "actions and institutional decisions which adversely affect..." minorities are made by a select group of dominant persons who harbor racist beliefs. Structural theory, on the other hand, asserts that the ideology is an integral part of the social system, and that the pivotal actions and decisions which lead to divergent outcomes are endemic in the social institutions of the racialized state (i.e., the United States). Neither theory, however, indicates the extent to which persons (i.e., discrimination theory) and/or social institutions (i.e., structural theory) must be saturated by the racial ideology, and how influential their discriminatory actions must be to create and maintain racial inequality. We are left to wonder: *Does the creation and maintenance of racial inequality require an abundance of racists or racialized social institutional actors? And to what extent do the bounded rational responses of "minority victims" alter the answer to the previous question?* Status construction theory is directly concerned with these issues and fills this theoretical void in the race theories.

Status Construction Theory

Ridgeway's (1991) status construction theory focuses on how "...nominal characteristics of people such as gender or race acquire status value in a society" (367). The theory centers on the micro-level social mechanisms whereby a status characteristic such as race may develop an independent reputation that is linked to social expectations about behavior, characteristics or attitudes (Ridgeway and Correll 2006; Ridgeway and Erickson 2000; Ridgeway et al. 1998; Ridgeway et al. 2009). Indeed, the theory does not give—nor was it designed to—an accurate accounting of the specific mechanisms that led to the

development of the concept of American race and the system of racial stratification. Status construction theory does, however, neatly outline the sufficient interactional (i.e., interaction based) mechanisms for the emergence and maintenance of status value (or belief) for a generic nominal characteristic.

Ridgeway (1991: p. 380) asserts that status beliefs emerge when a relationship develops between an exchangeable good and race (i.e., nominal characteristic) over the course of social interaction. This status belief may embody a norm regarding the expected behavior for a given race group. Alternatively, the belief may be in regard to an expected characteristic (i.e., productivity) of a race group that is readily used in various evaluations of ability/skill (i.e., statistical discrimination). In any case, the development of status beliefs about race implies that race begins to serve as a proxy for disparities in an important resource.

As for the specific interactional mechanisms that lead to the development of this norm of inequality, Ridgeway (1991) notes that:

...encounters among those that differ both in resources and the nominal characteristic actually create sufficient, consistent associations between the nominal characteristic and status beliefs to lead eventually to the creation of consensual beliefs about the characteristic's status value. (381)

Thus, status construction theory views the—potential—product of numerous interactions among persons of different racial groups with divergent resources (i.e., “doubly dissimilar” agents) as the emergence of a belief about the value of each race group—status belief (Ridgeway and Correll 2006; Ridgeway and Erickson 2000; Ridgeway et al. 1998).

Support for Ridgeway's theory is found in several social experiments (Ridgeway and Correll 2006; Ridgeway and Erickson 2000; Ridgeway et al. 1998; Ridgeway et al. 2009)—and a mathematical simulation (Ridgeway and Balkwell 1997). For example, Ridgeway and colleagues (1998) used two experiments to examine the emergence of status value on a nominal characteristic that was assigned at baseline. They found that “[after] two doubly dissimilar encounters, both the resource-advantaged [i.e.,

dominant] and, importantly, the resource-disadvantaged [i.e., subordinate] formed beliefs that people in the advantaged nominal category were higher in status” (347). This result highlights that the emergence of a status belief can occur quickly, and that subordinate group actors, like dominants, contribute to maintaining a status belief. Ridgeway and Erickson (2000) built on this research by testing varying conditions that actors can learn or spread status beliefs. They found that people can be taught status beliefs in the context of interactions—thus, one doesn’t need an initial resource difference—and, interestingly, learn the belief by witnessing interactions among doubly dissimilar persons. Most recently, Ridgeway and colleagues (2009) analyzed how gender contributes to the development of status beliefs and consequent actions based on the beliefs. They found that both men and women developed status beliefs in two encounters, but that women were much less likely to immediately act on the basis of the status distinction.

Altogether, Ridgeway’s theory and empirical research suggests that race gains status value in a given context when a relationship between resources and racial group becomes clear (Ridgeway 1991; Ridgeway and Correll 2006; Ridgeway and Erickson 2000; Ridgeway et al. 1998; Ridgeway et al. 2009). Her theory emphasizes the significance of initial resource differences between the groups, and highlights how “doubly dissimilar” interactions lead to the emergence of status beliefs for a nominal characteristic (i.e., race). Although her theory does not focus on discrimination/racism explicitly, Ridgeway’s research adds much-needed perspective to the aforementioned question: *Does the creation and maintenance of racial inequality require an abundance of racists or racialized social institutional actors?* Ridgeway’s theory and empirical research suggests that the creation of racial inequality needs enough racists/racialized institutional actors to markedly change the average outcomes of minority group members. Once changed, other actors perceive this shift as essential and we see the emergence of race as having status value. Status beliefs about race in the context will persist as long as the

relationship between resources/outcomes and racial groups confirm the widely held belief in ongoing micro-level interactions (Ridgeway and Correll 2006).

As for the second question, *to what extent do the rational responses of minority victims influence the creation and maintenance of racial inequality?* Ridgeway's theory and research suggest that minority group members do use the emergent beliefs to negatively inform their own behavior (i.e., lower resource demands in future interactions), and contribute to the level of racial inequality (Ridgeway et al. 1998). If the emergence of status beliefs about race are challenged by subordinates or dominants—through activism or another means—then the emergence of status distinctions are less likely (Ridgeway and Correll 2006).

Indeed, the concept of status beliefs as discussed by Ridgeway and colleagues is an integral part of structural race theory. Bonilla-Silva (1996: p. 476) notes that status beliefs are directly tied to a group's relative position in a social system and perform the ideological function of justifying stratification. The maintenance of race as a status distinction, then, is predicated on the persistence of a widely held status belief that is regularly confirmed by evidence in everyday social interactions (Ridgeway 1991). Structural race theory views this widely held, prejudicial—and sometimes seemingly non-racist—belief as an essential piece of the social structure that is used to maintain racial inequality (Bonilla-Silva 1996; Wellman 1977). Hence, Bonilla-Silva (2003) defines the “...racial structure as the *totality of social relations and practices that reinforce white privilege*” (9: emphasis in original). Although structural theory does not differentiate between actors/institutional agents who are overt discriminators and persons whose subtle prejudice (i.e., status beliefs) emerges or is maintained via doubly dissimilar interactions, this distinction represents an important connection that must be clarified if we aim to identify the source(s) of racial inequality. In the former case (i.e., overt racists) the beliefs are rooted in an essential, ideological notion of race dating back centuries, while the latter is a dynamic part of the modern social system operating in a given context that is, as suggested by contact theory

(Allport 1954; Dixon 2006; Pettigrew 1998), amenable to change through activism or other means. If we aim to understand, and eventually undermine, the system of racial inequality/privilege in the Western world, then we must highlight how the respective dynamic elements add to the maintenance of racial inequality in a specific context, and to the experience of race more broadly.

Methodology

Studying Dynamic Inequality

The discussion above highlights several mechanisms that contribute to the creation and maintenance of contemporary racial inequalities. Scholars, however, are left to wonder the extent to which the various mechanisms might contribute to racial inequality. At this juncture, research must begin to connect the theoretical discourse on the maintenance of racial inequality with the real world phenomena of race.

The critical issue for social scientists is that the aforementioned mechanisms are all dynamic parameters that are rooted in social interaction (Emirbayer 1997; Reskin 2003; Schwalbe et al. 2000; Tilly 1998; West and Fenstermaker 1995). Consequently, traditional quantitative methods that examine static forms racial inequality using cross-sectional survey data will not capture the dynamic social processes that are at play (Stewart 2008a, 2008b; Stewart and Ray 2007; Zuberi 2001).

One way to examine the dynamics of theories of racial inequality—and the emergence of many social phenomena—is agent-based modeling.³ These models represent rigorous, mathematical thought experiments which aim to unveil the emergence of social phenomena among agents interacting with each other using bounded rationality in their decision making (Epstein 2006, 2008; Epstein and Axtell 1996). Macy and Willer (2002) note that social scientists can “...understand [the] dynamics [of human groups] much better by trying to model them, not at the global level but instead as emergent properties of local interaction among adaptive agents who influence one another in response to the influence they receive” (144).

Although agent-based models are compelling methodological tools, they are not directly a product of a real world behavior or experimental responses as in the case of audit studies and social experiments, respectively. They do, however, allow social scientists to assess the plausibility of a theory by rigorously analyzing the emergence of a phenomenon thousands of times under different parametric circumstances (Centola et al. 2005; Epstein and Axtell 1996; Hanneman 1995; Leik and Meeker 1995). This theoretical advantage makes agent-based models of complex, dynamic phenomena an increasingly popular methodological tool among social scientists for studying formal theories—such as those that pertain to the emergence and maintenance of inequality.

There are many examples of agent-based research on social inequality, and a few that this paper directly builds upon. This body of research focuses on how boundedly rational agents with memories may locally interact to “grow social inequality” from the ground up. For example, Duong and Reilly (1995) used agent based models to grow social inequality using a symbolic interaction framework. They endowed simulated agents with an artificial neural network (i.e., memory) that allowed them to relate the physical features of other agents (e.g., race, suits) with their talent. These authors showed that various types of inequality/fads emerged in groups of agents that were often the result of “accidents.” Similarly, Axtell, Epstein and Young (2001) used a Nash bargaining game to study the emergence of class. They found that “...various kinds of social orders –including segregated, discriminatory, and class systems—can...arise through the decentralized interactions of many agents in which *accidents* of history become reinforced over time” (191; emphasis added). Although these simulations are extremely insightful, neither study highlights the specific mechanisms that shape the emergence of status beliefs. Rather, they suggest that chance accidents may lead to the development of inequality.

The development of status (i.e., inequality) is the focus of Ridgeway’s status construction theory. Ridgeway and Balkwell (1997) set out to assess the validity of one aspect of status construction theory—the diffusion of status beliefs—using mathematical simulations of group interactions. They

found that doubly dissimilar interactions are crucial to the development/diffusion of status beliefs, and that these beliefs eventually shape interactions of persons differing only on the nominal characteristic (i.e., singly dissimilar). They concluded that “...the major claims of status construction theory are logically consistent...the simulations we have reported add plausibility to the theory’s macro-micro analysis” (30). In this case, the authors shed considerable light on the plausibility of the theoretical mechanisms that are behind the emergence of widely held status beliefs. However, race theorists are left to wonder how discriminators and biased institutional actors may aid this process of status construction/maintenance in a given context.

The aforementioned agent-based research shows that agents operating with bounded rationality in local interactions can achieve suboptimal results (i.e., social inequality). Thus, the method has provided preliminary insights into how micro-level social dynamics can shape the experience of social inequality that is observed as a broader macro-phenomenon. However, there are many questions that remain: *How do overt discriminators contribute to the emergence of inequality and subtle prejudice? Does the emergence of inequality that is fueled by discriminators fade when discrimination is removed from the game? Does the subtle prejudice that emerges among subordinates (i.e., minority victimhood) contribute to inequality in the same way/magnitude as that which emerges among dominants?* These questions are the focus of the current analysis.

The Bargaining Game

To test the dynamics of the emergence and maintenance of racial inequality, I use an agent-based model of a Nash bargaining game (Nash 1950)[also see Axtell et al. (2001)]. The game involves agents randomly encountering one another and making a claim for a share of a specific good. As long as both agents do not combine to demand more than 100 percent of the good, they walk away with a share of the good. The game proceeds as follows. Agents begin the game endowed with race, age—which is uniformly distributed and ranges between 1 to α (i.e., maximum agent age in a simulation)—a small

amount of wealth that ranges from 50 to 140, a memory capacity with mean size k which ranges from $k-r$ to $k+r$ across agents in a simulation (k and r vary by simulation), and an initial bid that falls in the range of 10 and 90.

After receiving the initial endowments, the agents are randomly paired to bid on a good with a total value of 100. Agents are not aware of what the competitor will bid (i.e., agents make “blind bids”). Each agent in the pairing bids on her share of the good. If the sum of the two bids is greater than 100, then neither agent receives a payout. If, however, the sum of the two bids is less than or equal to 100, both agents receive their respective bid amount. Thus, if two agents interact and agent A bids 60 while agent B bids 30 units, then agents A and B will each earn 60 and 30 units of wealth, respectively, that will be added to their personal wealth holding.

In the course of this competition, each agent remembers their recent competitors’ bid. The agents use these memories to inform bids in future encounters. In the second encounter, then, agent A, from the example above, will bid 70 units because her last competitor only bid 30—her bounded, rational action is to assume the next competitor will bid 30 as well. For the third encounter, agent A would bid the average of 100 minus the average bid of her last two encounters. The agents have a maximum memory of k_i (i.e., k_A is the memory size of agent A) and use 100 minus the average bid in memory for as the bid for the ensuing encounters (i.e., 100 minus average competitor bids in memory).^{4,5}

After each encounter, a fixed number of units $[c]$ —which varies across simulations—is subtracted from each agent’s wealth holding. This subtraction represents the basic cost of life among agents. If an agent’s wealth falls below zero, the agent dies. A dead agent is replaced by newly born agent of the same race and discriminatory behavior (see below) with a random amount of wealth in the range of 50 to 140, a random memory capacity in the range $k-r$ to $k+r$, and a random initial bid that falls in the range of 10 and 90—all these random parameters are drawn from uniform distributions.

In addition to dying from lack of wealth, agents have a maximum age, α , which varies across simulations. When an agent reaches this age, they are replaced by a child. The child is an agent of the same race and discriminatory behavior of the parent, has a random memory capacity in the range of $k-r$ to $k+r$, and a random initial bid that falls in the range of 10 and 90. The child's wealth is a percentage, ρ , of the parent's wealth and this percentage randomly varies across simulations from 10 to 100 percent.

The bargaining game is played for N interactions between the 200 agents. The number of interactions, N , randomly varies (i.e., uniformly) across simulations and ranges from 30 to 260. At the end of the simulation, I use OLS regression to assess the magnitude and significance of racial differences in wealth holdings, average lifetime earnings per encounter, and average recent (i.e., preceding 10) earnings per encounter—I only present the results on earnings as they are very similar, but the wealth results are available upon request.

In certain runs of the simulation, I transform between 2.5 and 25 percent of the dominant racial group to become discriminators. These discriminators embrace Becker's (1971) "taste for discrimination." When they encounter a minority agent in the bargaining game, they automatically increase their bid by 1 to 20 units. Thus, a discriminating agent would always increase her bounded, rational bid to minority group agents. The point of inserting discriminators in the simulation runs is to assess their impact on the outcomes of subordinate agents, and the bids of other agents more broadly—discussed below.

In addition to discriminators, I also allow agents to invoke race-specific memories in certain simulations.⁶ Race-specific memories, here, refer to an agent making a bid in the current encounter based on: 1) the competitor's race, and 2) the average bid received by persons of that racial group in the past. Thus, in these simulations agents assess the competitor's race, then make a rational bid that is equal to 100 minus the average bid of competitors of that race group in their last k_i interactions—where

k_i is memory size of agent i . When agents have not recently interacted with a group (i.e., dominant) of agents and, consequently, do not have a race-specific memory to inform their bid, they make a uniformly random bid that ranges between 10 and 90.⁷

The bids based on race-specific memories allow me to assess two aspects of race theory that were discussed above. First, I assess the extent to which varying numbers of discriminators contribute to the emergence of subtle prejudice among dominant agents that leads to greater racial inequality. This is equivalent to examining the emergence of race as having status value among dominant agents and how this emergence contributes to racial inequality. The subtle prejudice, in this case, is born out of a small army of discriminators and presents itself as higher bids by non-discriminatory dominant agents when encountering subordinate agents in the bargaining game—and lower earnings of subordinates. An important caveat in models with race-specific memory is that discriminators use the maximum of the discriminatory bid (i.e., using bounded rationality without race specific memory) and race-specific bid. This caveat places a maximum on how large disparities can become, and jibes with the simple notion that discriminators become satisfied once inequality reaches a certain level—and then bid purely on the basis of bounded rationality with race-specific memory.

The second utility drawn from race-specific memories is that I assess the extent to which minority victimhood contributes to racial inequality. Here, minority group agents also use race-specific memories to inform their bids. Given that the average expected bid by a dominant competitor would be higher when discriminators discount the average minority bid, the average minority bid when invoking race-specific memories in an encounter with a dominant agent would be lower. Thus, minority victims contribute to the level of racial inequality in the social system.⁸

Altogether, I ran 2,500 iterations of the bargaining game where 200 agents engaged in N ($30 \leq N \leq 260$) bargaining encounters with other agents. The simulation parameters varied considerably across the simulations. Table 1 contains the mean and range of each simulation parameter. As one can

see, there are a host of factors that *independently* vary across the simulations. These include agent life expectancy, the percent of dominant discriminators, the magnitude by which discriminators increase bids, overall discrimination (i.e., the product of percent of discriminators and magnitude of discrimination), the percentage of subordinate group agents, memory length, random noise in memory length across agents, random noise in agents' bids, the cost of maintaining life, the percentage of parental wealth inherited by children, the number of bidding interactions of each agent (i.e., rounds in game), the percentage of interactions through which discriminators can operate (i.e., assuming a law inhibited discriminators from discriminating after a specific encounter), the use of race-specific memory among dominant group agents, the use of race-specific memory among subordinate group agents, and the existence of racial disparities in wealth at the beginning of the simulation. Variation in these parameters—which are all independent and randomly, uniformly distributed⁹—allows me to assess how each contributes to the emergence and maintenance of racial inequality in a social system. For this paper, I am particularly interested in how the number/magnitude of discriminators contributes to the development of racial inequality, and how the emergence of subtle prejudice among dominant agents and the actions of minority victims exacerbate the effects of discriminators.

(Table 1 About Here)

Analytic Outline

I present the analytic results of the bargaining game in two steps.¹⁰ First, I use figures to highlight how discrimination, subtle prejudice and minority victimhood lead to the emergence of group disparities in bids and earnings over the course of 125 rounds of the game. Then, I use multivariate regression to highlight the parameters that contribute to the emergence of group inequality in 2,500 unique trials of the bargaining game. The latter analysis is extremely robust as it incorporates a high degree of stochasticity into the assessment of how the aforementioned processes dynamically contribute to the development of inequality. Let us now turn to the Nash Bargaining game analyses.

Results

Simulation Analysis I: Dynamics of Inequality

Figures 1A through 1E depict the group differences in average bids and earnings for 125 rounds of the bargaining game. The average values in the respective panels are taken from 50 simulations with parameters randomly varying for: 1) maximum agent age, 2) percentage subordinate agents, 3) agent memory size, 4) random noise in memory across agents, 5) noise in agent bids, 6) percent of wealth inherited by children, and 7) costs of maintaining life. The variation in these parameters mirrors that shown in Table 1.

(Figure 1 about here)

Figure 1A shows that in a world without discrimination and where agents do not use race-specific memories to inform bids, the average differences in group bids and earnings are nearly synonymous. As one would predict [see Appendix Equations A1 and A2, or Note 5], the bids of each group quickly converge to a mean of 50 and the earnings of each group both reach their maximum of 41 in the 125 iteration of the bargaining game (not shown). Indeed, there remains a noticeable amount of noise in the group difference statistics. However, there is no systematic difference that emerges across the 125 rounds of the game.

In Figure 1B, I designate approximately 10 percent of dominant agents as discriminators who increase their bids to subordinate agents by 10 points—this is equivalent to all dominant agents increasing their bids to subordinates by 1 point [see Appendix equation A3 for more discussion]. Although the difference in bids varies considerably, there is a modest, significant trend that favors the dominant group. Hence, the average difference in bids across the simulations is -0.538 which creates a significant average earnings disparity of -1.465 across the bargaining games. This result suggests that direct, overt discrimination has a modest effect on creating group disparities, and that it would take a

fairly large amount of discrimination to increase earnings disparities to 5 to 10 units across the simulation (i.e., 35 to 70 percent of dominants increasing bids to subordinates by 10 points).

In addition to discrimination, the data in Figure 1C incorporates dominant agents using race-specific memories to inform their bids in the model. This addition leads to the emergence of two unique phenomena. First, the average bid of subordinate agents decreases to approximately 1.2 units lower than that of dominant agents. This increase in the bids of dominant agents is attributable to a two-step process where: 1) the discriminators modestly drive down the bids of a few subordinate agents, and 2) non-discriminating dominants begin to make “smart” bids¹¹ as a result of receiving lower bids by subordinate competitors. The second point of interest is that dominant agents’ earnings outpace subordinates by an average of 3.4 units—and a maximum of 4.4 at the end of the simulation.

Figure 1D incorporates subordinate memory, as opposed to the dominant memory in panel C. The striking finding in this figure is that while the average bids of subordinate agents decrease to slightly less than one unit below that of dominant agents (-0.840), the group earnings disparity quickly emerges and stabilizes at approximately 5 units. The fairly small bid difference is a product of subordinate agents increasing their bids to other subordinates. This small difference in bids, though, pales in comparison to the sizeable, significant earnings disparity that emerges across the simulation.

The final panel, Figure 1E, portrays the emergence of inequality in a world with discrimination and allows all agents to use race-specific memories to inform their bid. The average group difference in bids varies considerably across the 125 rounds of the game. There is, however, a sizeable average difference in bids of 2.2 units in favor of dominant agents. In addition to the group difference in bids, there is a significant disparity in average earnings that appears shortly after the game begins. The difference in average earnings—which has a mean of 5.5 and maximum of 6.7—is significantly larger than in the simulations where only one group uses memory to inform bids. This fact, again, suggests that dominant and minority memory—subtle prejudice and minority victimhood—may each make

unique contributions to group inequality. Unfortunately, the current graphic analysis does not allow me to disentangle the respective contributions to inequality. I now turn to the multivariate regression analyses to assess the extent to which discrimination, subtle prejudice and minority victimhood, as well as other simulation parameters contribute to the emergence of group inequality in the bargaining game.

Simulation Analysis II: A Statistical Assessment of Inequality

The preceding analysis showed that group disparities emerge from a few mechanisms. The average disparities shown in Figures 1A through 1E, however, do not reveal how each of the simulation parameters contributes to “growing inequality.” I use ordinary least squares regression to assess how these factors contribute to the emergence and maintenance of group inequality.

Table 2 shows the results of the regression of the average earnings disparities—as shown in Figure 1B—on discrimination (i.e., these simulations do not incorporate race-specific memory into bids). The results—based on a subsample of 607 simulations that do not incorporate race-specific memory—indicate that discrimination operates to increase group earnings inequality. A one unit increase in discrimination leads to an increase in group earnings disparities by 0.63, and accounts for approximately 18 percent of the variance in observed inequality in this subsample of simulations. Models 2 and 3 reveal that the number of rounds, number of rounds without discrimination, proportion of subordinate agents, memory length, life expectancy, and the magnitude of random noise in agents’ bids all significantly contribute to group inequality. More bargaining rounds, longer memories, and longer life expectancy in a simulation operate to increase group inequality. In contrast, a longer concluding period without discrimination, more bid noise, and larger share of subordinate agents all operate to decrease inequality. Notably, the latter result on proportion subordinate—which has a huge impact on the r^2 statistic—and the significance of discrimination both concur with the expected relationship shown in appendix equations A3 and A4 (see Appendix A).

(Table 2 about here)

Table 3 shows the results of the full model where discrimination is present and all agents use race-specific memories to inform their bids—the use of memories by subordinates and dominants vary independently in these simulations. Model 1 parallels the results of seen in Figure 1B and Table 1. Specifically, discrimination operates to modestly increase disparities in the bargaining game—10 percent of discriminators increasing bids by 10 points would lead to a -0.70 point difference in average earnings across the course of a simulation. Dominant and subordinate memory, however, markedly contribute to the amount of inequality in the bargaining game. Models 2 through 4 reveal that each type of race-specific memory (i.e., dominant and subordinate) operates to significantly widen the average earnings disparity by approximately 3 points.¹² This gap remains after controlling for a host of other simulation related characteristics.

(Table 3 About Here)

As in the discrimination only analysis [shown in Table 2], a significant contributor to the model of average earnings inequality across the simulations is percent subordinate—the jump in the r^2 statistic from 26% to 53% is largely a result of this variable.¹³ An increase in the percentage of subordinate agents operates to decrease inequality. This variable is especially important in adding to simulations where dominant agents are using race-specific memories to inform bids (results not shown, available on request). In addition to the proportion of subordinate agents, an increase in the number of concluding bargaining rounds without discrimination and random noise in agent bids both operate to decrease the level of inequality in the bargaining game.

The results in thus far show that both discrimination and, more so, race-specific memories drive the emergence of inequality [Figure 1] and the level of average inequality observed across several dozen rounds of a simulated bargaining game [Tables 2 and 3]. However, they do not highlight the effect of these factors on the maintenance of inequality. Table 4 contains the results of the multivariate analysis of “recent” earnings in the simulated Nash bargaining games. Recent earnings, here, refers to the

average earnings an agent received—and earnings disparity observed—in the final 10 rounds of a game. These results are very similar to those seen in Table 3 which focuses on average earnings disparities across the entire game. Specifically, discrimination and race-specific memories all contribute to the level of inequality at the end of a bargaining game simulation. The effect of discrimination, though, is markedly smaller in these models. This results from the fact that discrimination was discontinued before the final 10 rounds of the bargaining game in nearly 80 percent of the simulations. The coefficients on the race-specific memory variables are still large and significant. These variables, along with group composition, operate to maintain inequality in bargaining games even when discrimination is no longer being used (i.e., there are no big bad racists driving inequality).

(Table 4 About Here)

The latter point is best seen in Table 5 where the multivariate regression analyses are stratified by bargaining rounds since discrimination ceased. The simulation data, here, are stratified into four groups where discrimination ended (A) less than 10 rounds, (B) 10 to 50 rounds, (C) 50 to 100 rounds, and (D) greater than 100 rounds before the conclusion of the game. These groupings allow one to assess the role of discrimination and race-specific memory in shaping inequality when discrimination is present (i.e., Model A) and long after it's discontinued in the social system (i.e., Model D). Model A, less than 10 rounds without discrimination, shows that discrimination has a large, significant impact on recent earnings when it is still operating. Likewise, race-specific memories also play a major role in recent earnings disparities with dominant and subordinate memories each increasing disparities by approximately 4 units. When discrimination is removed from the system, though, it does not have a significant, long-lasting effect—the discrimination coefficients in Models B through D are not significant and they are all significantly different from that in Model A. The variables that operate to maintain inequality in the bargaining game when discrimination is not present are race-specific memories and percent subordinate—none of the race-specific memory coefficients are significantly different than each

other. Thus, subtle prejudice and minority victimhood are two simple, practical mechanisms for the maintenance of inequality in a world without any overt discrimination (i.e., big bad racists).

(Table 5 about here)

Summary of Results

Altogether, the results above indicate that discriminators, subtly prejudiced dominants and minority victims all contribute to racial inequalities in the simulations. Although discriminators play an instigating role in models with race-specific memories, they do not make an appreciable difference otherwise.

Race specific memories—subtle prejudice and minority victimhood—do the heavy lifting in creating and maintaining racial disparities in agent earnings. Dominant agents using race-specific memories quickly learn that subordinate bids, as a consequence of discriminators, are lower than dominants. Likewise, minority agents using race-specific memories rapidly learn that dominants bid higher than subordinates, and begin to make lower bids to dominants. These two processes feed on each other to “grow” racial inequalities in earnings.

Discussion

The question remains, *how many racists does it take to create and maintain racial inequality?* Unlike other more humorous questions in this vein, the answer to this question is quite complex. There are no ladders to be held, light bulbs to be turned or funny punch line answers. Rather, the answer must bridge the divide between micro-level interactions and macro-level phenomena, and shed light on the host of dynamic mechanisms that contribute to inequality. The current paper answers the question using agent-based modeling. This “complex” approach allowed me to examine how various parameters contribute to the emergence and maintenance of racial inequality in an artificial society of social agents. Furthermore, it allowed me to refine the simple question so as to address more recent theoretical arguments about the sources of racial inequality. Hence, the refined question of interest: *Can a small*

number of big bad racists create group disparities via the emergence of subtle prejudice and minority victims?

The results here support the notion that a small group of discriminators can inspire subtle prejudice and minority victims to emerge which jointly drive the creation of racial inequality. Once established, the maintenance of the system of inequity requires no discriminators—when we remove discriminators after several rounds of the game, significant disparities remain. Thus, discrimination, like a good leadoff batter in baseball, “puts men on base,” while subtle prejudice and minority victims “drive in the majority of the runs.” Although discrimination plays a minimal role in the models where agents invoke race-specific memory, all three factors—discrimination, subtle prejudice and minority victims—contribute to racial inequality in the simulations.

In addition, the results reveal that racial composition uniquely influences the level of observed inequality in a system. Specifically, the multivariate results indicate that the level of racial inequality in a simulation declines considerably as we increase the percent of subordinates in the population. The mechanism driving this result is best explained using Ridgeway’s (1991) notion of “doubly dissimilar” interactions. When there are only a few subordinate agents, discriminators are able to have a larger relative impact (i.e., each discriminatory interaction affects an appreciable share of the subordinate population when their population numbers are small). This larger relative impact inspires more “doubly dissimilar” interactions as subordinates are then more likely to demand less from dominants which, consequently, creates subtle prejudice (i.e., racial stereotypes) among non-discriminatory dominant agents.

When there are a large number of subordinate agents in a simulation, the impact of discriminators is smaller (i.e., each discriminatory interaction affects a small share of the subordinate population when their numbers are large). Again, Ridgeway’s (1991) doubly dissimilar interaction mechanism is key. A larger subordinate population size implies that non-discriminatory dominant

agents are interacting with an array of subordinates who, in this case, are less likely to make lower bids because they are less likely to suffer from discrimination. Thus, a large population of subordinates provides more opportunities for singly dissimilar interactions (i.e., different race, same bid) which undermine the emergence of subtle prejudice.

Altogether, the results here suggest that a few actors/institutional actors that use race to unfairly distribute rewards and resources can lay the foundation for the emergence and maintenance of widespread racial inequality. The results further underscore the need for a broad, multifaceted policy approach to eradicating racial inequality because non-discriminating agents—dominant and subordinate—likely make unique contributions to the experience of racial inequality. Policies that strictly focus on agents expressing discriminatory sentiments will likely miss the mass of individuals whose behavior constitutes the system (i.e., structure) of racial inequality.

Indeed, some readers may suggest that we should not give considerable weight to these results and policy insights as they are only the product of a mathematical simulation—a model. Epstein (2008) writes of this assertion:

Anyone who ventures a projection, or imagines how a social dynamic—an epidemic, war, or migration—would unfold is running some model. But typically, it is an implicit [theoretical] model in which the assumptions are hidden, their internal consistency is untested, their logical consequences are unknown, and their relation to data is unknown. ... The choice, then, is not whether to build models; it's whether to build explicit [agent-based] ones. In explicit models, assumptions are laid out in detail, so we can study exactly what they entail. On these assumptions, this sort of thing happens. When you alter the assumptions that is what happens. (p. 1.3-1.5)

The benefit of mathematical simulation, then, is that one can test the validity, sensitivity and scope of theory by trying to model a social phenomenon from the ground up using an array of logical assumptions. Mathematical simulations of racial inequality allow scholars to see *how* the micro-level theoretical dynamics of race may work to shape/maintain disparities at the macro-level.

In truth, most mathematical simulations do not *directly* represent a specific social system [for example see Axelrod (1984), Axtell et al. (2001), and Centola et al. (2005)]. They do, however, broadly

represent an array of social situations and highlight how phenomena may emerge in the respective social domains. In the current case, the model—the Nash Bargaining Game—broadly represents an array of social arenas where the distribution of valued resources is a key activity. The U.S. educational system is one such arena. Our model suggests that a few discriminatory agents that ration educational resources on the basis of race may inspire a system of inequality.

For example, suppose a few racially biased teachers/school systems give black students less attention and resources than white students. These “discriminating agents” would lead black and white students/families to develop distinct micro-level expectations regarding the amount of goods (e.g., attention, resources) they deserve. These distinct expectations would then shape the behaviors and demands of students/families and, consequently, lead to disparities in achievement—a direct result of racial differences in demands for scarce resources in educational settings. Thus, educational disparities can emerge from a few discriminatory actors/school systems which inspire boundedly rational persons to behave in ways that augment racial inequalities in achievement. And, most importantly, this process could unfold in a series of seemingly unimportant micro-level interactions that do not undermine a group’s larger beliefs about fairness, hard work and meritocracy.

Conclusions for Future Research

The agent-based model presented here is far from being the final word on race theory or the key explanation for racial inequality. Rather, it is a first step in creating a multi-method (e.g., qualitative, social experiment, etc) body of research that focuses on how dynamic processes contribute to racial inequality. Future empirical research on the dynamics of race—by this and other authors—can build on this paper by further examining: 1) the extent to which subtle prejudice exists in various social dimensions, 2) the degree to which subtle prejudice may be undermined through singly different interactions in a given social environment, and 3) how subordinate actors operationalize victimhood

(i.e., stereotypes) in various social domains. This research will bring light to those interpersonal mechanisms that contribute to racial inequality and how they operate in real world situations.

In addition to empirical research, scholars should increasingly turn to mathematical simulations to study how other parameters may uniquely affect racial inequality. For example, Ridgeway's (1991) model suggests that the resource gap that inspires the emergence of status beliefs is related to network structure [also see Royster (2003)]. Specifically, network structure is argued to be characterized by homophily and, consequently, limit the number and nature of contacts across racial groups. This theoretical assertion suggests that changes in network structure should affect the emergence and maintenance of inequality. Thus, studies on how network structure is related to varying levels of racial inequality will greatly add to the literature. Other possible lines of research using agent-based models involve studying: 1) how anti-racists affect the emergence of racial inequality, 2) whether other forms of status (i.e., gender) affect the level of racial inequality in a system, and 3) how disparities in treatment (i.e., discrimination) compound across interactions such that the source(s) of difference become embedded in other, non-discriminatory variables (i.e., subordinate bids). These increasingly complex models of inequality will greatly inform our understanding of how race shapes countless interactions and, subsequently, life chances more broadly.

In sum, agent-based models are one of many tools that researchers should increasingly use to understand how the macro-system of racial inequality sustains itself through the micro-level social interactions of everyday persons. This type research must supplement other forms of empirical/theoretical research to form the basis of a larger research program focusing on the interpersonal and structural mechanisms that shape racial inequality. Together, a multifaceted research program—such as that seen in group processes research [see Szmataka, Lovaglia and Wysocka (2002) for examples]—that examines the impact of boundedly rational micro-level behaviors on racial inequality using simulations, the behaviors of everyday people using social experiments, audits and

qualitative methods, as well as the relationships between race and other variables in large-scale survey data will yield better insights on the source(s) of racial inequality than existing programs which strictly use one method. This type of broad research program will further unveil how dynamic theories intersect to shape observed disparities in social outcomes, and reveal the policy mechanisms through which we may eventually undermine and eradicate racial inequality.

Appendix A: Mathematical Analysis—The Bargaining Game

We can write the expected bid (y_i) of agent i in the Nash bargaining game with no discrimination using a simple set of equations based on memory size (k), the number of the encounter (n) and the average bid of past competitors. It is written:

$$y_{in} = \begin{cases} U(10,90) & \text{for } n = 1 \\ [100 - \overline{Bid}_{ci}] & \text{for } n \geq 1 \end{cases} \quad (A1)$$

where \overline{Bid}_{ci} is the average bid by competitors to agent i prior to interaction n . We estimate this as:

$$\overline{Bid}_{ci} = \begin{cases} \frac{\sum_{j=1}^k Bid_{cj}}{n} & \text{for } k \geq n \\ \frac{\sum_{j=n-k}^n Bid_{cj}}{k} & \text{for } k < n \end{cases} \quad (A2)$$

where Bid_{cj} is the bid by competitor to agent i in the respective interaction. Together, equations A1 and A2 show that the first bid is a shot in the dark—a random, uniformly distributed bid that ranges between 10 and 90. All subsequent bids are based on a rational response to a set of recent competitor bids (i.e., bounded rationality). One can show that this simple model (i.e., with non-specific memory and without discrimination) will converge to an average bid of 50 units fairly quickly—the average random competitor bid approaches 50 as n increases.

When we include discrimination in the model, the expected bids of agents change markedly. The bids change on the basis of the percentage of discriminators (D), the magnitude of discrimination (A) and the percentage of minorities in the game (π). Using these terms, we write the expected bid for a dominant group member as:

$${}^{Dis}y_{in}^D = \begin{cases} U(10,90) + D \cdot A \cdot \pi & \text{for } n = 1 \\ [100 - \overline{Bid}_{ci} - D \cdot A \cdot \pi] & \text{for } n > 1 \end{cases} \quad (A3)$$

Where the “Dis” term in upper left quadrant of y indicates the bid is under a discriminatory regime, and the D term in the upper right quadrant of y refers to the bid being the average for a dominant agent.

This equation indicates that the expected bid of a dominant is increased by the value of the product of

D , A and π —the percentage of discriminators, the magnitude of discrimination and the percentage of minorities. The average dominant agent, then, will increase their average bid by the value $[D \cdot A \cdot \pi]$ when discrimination is present. Readers should note that although every dominant agent is not a discriminator, the average dominant agent in the game has a modest amount of discrimination and will increase her bid accordingly (as represented in the product of the latter terms).

The average minority bid will also change when discriminators are included in the model. In the case of minorities, the average bid is reduced by the same amount.

$${}^{Dis}y_{in}^M = \begin{cases} U(10,90) & \text{for } n = 1 \\ [100 - \overline{Bid}_{ci} - D \cdot A \cdot (1 - \pi)] & \text{for } n > 1 \end{cases} \quad (A4)$$

Minorities, then, begin the game bidding as usual. They then reduce their bid by the factor $[D \cdot A \cdot (1 - \pi)]$ in subsequent encounters based on their interaction with dominant group agents.

Equations A3 and A4 show that when discrimination is present it operates to increase the average dominant agent's bid and, consequently, reduce the average minority agent's bid. The impact of discrimination in a given encounter will likely be small as it is the product of three factors—two of which (i.e., D and π) may be quite small. Thus, one should expect that a small number of overt discriminators can have a modest direct impact, at best, across a simulation game.

When we allow dominant and minority agents to invoke race-specific memories, the expected bids of the respective groups change considerably. The expected dominant bid with memory in the first and second encounters of the game are the same as seen in Equation A3. In subsequent encounters, however, the expected bid incorporates the response bids of minority competitors and compounds the level of discriminatory treatment. We write expected bids of dominants as

$${}^{RM}y_{in}^D = \begin{cases} U(10,90) + D \cdot A \cdot \pi & \text{for } n = 1 \\ [100 - \overline{Bid}_{ci} + (n - 1) \cdot D \cdot A \cdot \pi] & \text{for } n > 1 \\ [100 - \overline{Bid}_{ci} + [A \cdot \pi]] & \text{for } n \rightarrow N \end{cases} \quad (A5)$$

where the “RM” term on y refers to the fact that the bid invokes race-specific memories. The invocation of race-specific memories leads to a higher level of discriminatory treatment—the average bid is increased by the product $D \cdot A$ when facing a minority group agent in the first two rounds, and increases arithmetically as more dominants encounter minorities that have been discriminated against. Thus, invoking race-specific memory leads to discriminatory bids compounding across encounters. As subordinate agents increasingly interact with discriminators, their average bids will decline. Dominant group members will then use these inferior bids to inform current bids (i.e., subtle prejudice) and, consequently, build on the disadvantages created in earlier encounters.

Dominant bids will reach a maximum discriminatory level when the product of the terms $[(n-1) \cdot D]$ equals 1—when bids to subordinates are, on average, A points lower than those given to dominants. This point represents the line at which discriminatory agents realize their aim (i.e., a group difference in bids that is equal to A) and cease their pushing for greater inequality through higher bids to subordinates. This point, however, is a moving target as different discriminator agents will reach/perceive this maximum in different intervals—one can view this as multiple discriminatory agents each taking their foot off their respective gas pedals at different times in an effort to maintain a desired group speed in relation to the finite group of competitors that they each have information about.

When minority agents are invoking race-specific memories in their bids, the initial bid also remains the same as was seen when discriminators are present—a uniform bid between 10 and 90 units. After the first encounter, the bids change markedly.

$${}^{RM}y_{in}^M = \begin{cases} U(10,90) & \text{for } n = 1 \\ [100 - \overline{Bid}_{ci} - (n-1) \cdot D \cdot A \cdot (1 - \pi)] & \text{for } n > 1 \\ [100 - \overline{Bid}_{ci} - [A \cdot (1 - \pi)]] & \text{for } n \rightarrow N \end{cases} \quad (A6)$$

Minority bids, when invoking race specific memories, are influenced by the average bids they receive from dominants. When bidding against another minority agent, minority agents play the game normally and do not change their bid. The bid for a dominant agent after the first encounter, in contrast, is

reduced by the factor $[(n-1) \cdot D \cdot A]$. The reduction of the minority bid against dominant competitors appears to be very similar to the general reduction in the minority bid when discriminators are present—without race specific memories (Equation A4). When race specific memory is invoked, though, the bid is reduced by a much larger amount when facing dominant agent (i.e., by a factor of $(n-1)$). This increasing reduction is a product of subordinates facing an increasing number of high bids from dominants—as dominants invoke subtle prejudice, the average bid to subordinates increases considerably.

Altogether, this formulaic analysis reveals three things. First, if discrimination is not operating the average bid of both groups will converge to 50. Second, discriminators will lead to small, incremental changes in the bids to minorities and, consequently, to reductions in the average minority bid across encounters. The amount of change, however, will likely be small unless there is a preponderance of discriminators in the game or the magnitude of discrimination is exceptionally large. The final insight from the mathematical analysis is that invoking race-specific memories will generally lead to much higher bids from dominant agents to minority competitors. Additionally, minority agents invoking race-specific memories will respond to dominants by lowering bids. This raising and lowering of bids on the part of dominants and subordinates, respectively, in the race-specific memory model will compound across interactions as discriminators increasingly operate to lower the average bids of subordinates.

Notes

¹ Indeed, a third reason for the decline in discrimination theory is that neoclassical economic theory suggests that discrimination is not a rational action in labor—or other—markets (Arrow 1971; Becker 1964, 1971; Varian 1992).

² Another theory which articulates minority disinvestments as being a component of racial inequality is Steele's (2003) stereotype threat. This theory and associated research indicates that minorities often underperform out of a fear of reinforcing a negative stereotype. Although the minority is not making a conscious disinvestment in the case of stereotype threat theory—as we see in other theories—minority actors do contribute to the observed level of inequality by lowering outcomes in a specific encounter which may compound across encounters.

Though less intuitive, one can also look at relative deprivation and distributive justice theories as representing victim theory. In both theories, agents who perceive they did not receive their just reward in the goods allocation process may disinvest in skills/resources (Jasso and Rossi 1977; Jasso and Resh 2002; Runcimann 1966; Stewart 2006; Taylor 2002).

³ There are a two other ways to quantitatively test the theoretical dynamics of racial inequality. First, one can use a social experiment. The experiment, in this case, would involve having dozens of respondents of different race groups interacting with other actors through a computer to see if persons expressed racial bias, the extent to which subtle prejudice develops as a result of other actor's discrimination, and if/how minority respondents contribute to racial inequality. The aforementioned studies by Ridgeway and colleagues and the work on stereotype threat by Steele (2003) are great examples of social experiments. In Ridgeway's case, the researchers always found that a difference in resources and/or a series of social interactions that implicitly highlight status distinctions leads to the emergence of beliefs about the competence/resources of persons in a certain nominal category—and that people act on this belief very quickly (Ridgeway and Correll 2006; Ridgeway and Erickson 2000; Ridgeway et al. 1998; Ridgeway et al. 2009). Although this research suggests that the emergence of status beliefs requires an initial resource difference and/or biased behavior of actors, it does not indicate how overt discrimination (i.e., behavior) by a small group of actors/institutions may lead to the development and maintenance of racial inequality more broadly. Furthermore, the social experiments may not reveal exactly how the behaviors/beliefs emerge in the wild.

A second method one can use to study the theoretical dynamics of race is audit studies—or vignettes. Audit studies involve sending pairs of actors—or descriptions of actors (i.e., vignettes)—of different racial groups to see whether respondents treat the auditors in a discriminatory fashion. Pager (2001) and others (Bendick, Brown, and Wall 1999; Bendick, Jackson and Reinoso 1994; Bendick et al. 1991; Fix, Galster & Struyk 1993; Heckman and Siegelman 1993; Loring and Powell 1988; Saltman 1979; Yinger 1993) have used audit studies to convincingly show that discriminatory behavior is a norm in various social environments. While audit studies allow social scientists to examine behaviors in the real world, they are quite limited. More specifically, audit studies are only able to gauge the behavior of an actor in a single, discrete interaction and, consequently, fail to highlight how this behavior may uniquely intersect with other behaviors to shape the emergence and experience of racial inequality.

⁴ In addition to using this mean strategy (100 minus average of competitor bids in memory), I also examined the game using three other strategies. The first two strategies involved agents using the (1) mode and (2) median of competitor bids. In these cases, the agent's bid was 100 minus the mode/median of recent competitor bids. I also examined the game using a (3) probabilistic strategy where agents would increase/decrease their bid in response to a specific success/failure rate—which also varied across simulations. The results of these strategies differed greatly. Specifically, the results for simulations using mode and median strategies are directly in line with those presented here. In contrast, the probabilistic strategy showed considerably less inequality than seen among the other strategies. This was a product of the slower learning rate associated with this strategy. When using the probabilistic strategy, agents don't automatically move to the boundedly rational bid, but slowly move towards it based on success/failure rates (readers should note that this slow learning of boundedly rational bids in a game about distributive justice sharply contrasts that seen in Ridgeway's experiments discussed earlier). To assess the value of the probabilistic strategy, I used an evolutionary algorithm where I placed agents using the respective

strategies in competition with one another in the simulations (see Holland (1972, 1995) for a discussion of evolutionary algorithms). The results from this evolutionary model revealed that the probabilistic strategy was quickly overtaken by the other three in a handful of generations.

⁵ Although I use a simulation and allow agents to solve for optimal bid through interactions with other agents, one may also solve this problem in classic game-theoretic form. Specifically, the Nash Bargaining game can be written as a two player normal form game $G = \{S_1, S_2; u_1, u_2\}$ where S_i refers to the strategy space for player i , and u_i refers to the payoffs associated with various strategies. The payoffs associated with the game can be represented as a two-way table (i.e., 9x9) where the rows and columns represent the specific strategy chosen by players 1 and 2, respectively. The payoff table is characterized by multiple Nash Equilibria (i.e., they are spaces where neither player will deviate from their strategy) where the bids of both players sum to 100. If we assume, however, that each player believes the other to be playing a uniformly random mixed strategy (i.e., choosing a bid by uniformly picking values in the distribution 10, 20, ..., 80, 90), then the Nash equilibrium will be perfect strategy response bid of 50—or a normally distributed mixed strategy with a maximum probability at the point of 50. This bidding strategy will return an average of 27.8 points per encounter and there is no mixed response that beats this pure strategy response to a competitor using the uniform mixed strategy. Thus, a rational agent would infer that 50 is the best response bid when facing a competitor(s) using a uniformly mixed strategy.

Indeed, in a world of competitors who use different strategies, this may not always be the best response. If we take a simple case—which we build on here—where dominants are discriminators and shift their strategy from a uniform mixed strategy on the interval 10 to 90 to the interval 30 to 90, then we can show that the best response of subordinates to this shift is a pure strategy bid of 40. In turn, the best response to this pure strategy bid by subordinates is a response bid of 60 by dominants. Thus, when subordinates believe dominants are using a biased mixed strategy, it induces/reveals a pure—or unique mixed—strategy response that readily leads to biased bids by dominants (i.e., a self-fulfilling prophecy). One can also solve this inversely showing that when dominants believe that subordinates are using a deflated, mixed strategy, the pure strategy response is a racially biased bid.

⁶ The results below focus on the distinction between simulations where either “no” agents or “all” agents of a group use race-specific memories—I do allow them to vary by group across the simulations such that dominants may use race-specific memory when subordinates do not. Indeed, these results do not highlight how the percent of agents using race-specific memory in each population contributes to inequality in the bargaining game. I did run 2,500 additional simulations where I independently varied the percentage of subordinate and dominant agents using race-specific memories. The results perfectly mirror those shown here and appear in Appendix Tables B1 and B2.

⁷ This is equivalent to playing a uniform strategy where the agent randomly selects a bid on the distribution. Indeed, one can show using a normal-form solution that the best counter to this strategy is a bid of 50 (i.e., the expected bid of someone using this strategy is 50). In the current case, however, I allow the agents to “learn” the optimal bid through interaction and only assume agents have a simple strategic architecture—agents are not computing the rational play of their competitors as in game theory.

In addition to using the uniform strategy when encountering an unfamiliar competitor, I also ran simulations where agents used 100 minus the average bid of other competitors. The results of these simulations have the same form of those shown here, but are of a smaller magnitude (i.e., the effect of race specific memories are approximately 33% smaller). The factor that drives this difference is an accumulation of agents “treating” unfamiliar competitors like they are members of the group which readily undermines the amount of privilege/inequality in the system.

⁸ In addition to assessing the two aspects discussed here—subtle prejudice and minority victimhood—I also examined the impact of different subordinate responses/strategies to race-specific memories on the level of inequality in the Nash bargaining game. Specifically, I ran 2,500 additional iterations of the simulation (not shown, available upon request) where I programmed subordinate agents to randomly use one of five responses to the race-specific memories information in each run. The five responses included: 1) do not use race-specific memory in bid determination (i.e., fair bids), 2) use the past group averages to determine bid (minority victimhood, which is

discussed/analyzed here), 3) use the subordinate group average in determining all bids (i.e., 100 minus the subordinate average for all competitors), 4) use the dominant group average in determining all bids, and 5) use the dominant group average in determining bids for subordinates (i.e., 100 minus dominant average becomes bid for subordinates) and the subordinate group average in determining bids for dominants (i.e., 100 minus the subordinate average). As one can see, the first two responses/strategies parallel those used in the simulation presented here. The third and fourth responses/strategies use race-specific memory, but do so in a uniform way such that all agent encounters are “treated” using the average of encounters with the subordinate or dominant group, respectively, in bid determination. The last response/strategy is an anti-prejudice variety which inverts the race-specific information (i.e., averages) used in bid determination in an effort to undermine inequality.

The results of the analysis using these different responses/strategies indicate that the strategy that significantly adds the most to inequality in the game is the third (i.e., use subordinate average) which has roughly twice the impact of the second variety (i.e., minority victimhood) which is examined here. The other two strategies—use dominant average (4) and use anti-prejudice (5) stance—also significantly add to the amount of inequality in the system. These latter strategies, however, have a significantly smaller contribution to the overall level of inequality in the game than the second strategy (i.e., minority victimhood) which is analyzed here. The subordinate strategy that performs best is the first (i.e., fair bids) where the agents do not use race-specific memories in bid determination.

⁹ The random uniform distribution for the variables was created using the random number generator in Matlab 7.8. To ensure a random—and replicable—distribution, I also varied the seed across the simulations.

¹⁰ In addition to these two analyses, I performed a mathematical analysis of the Nash bargaining game that appears in Appendix A.

¹¹ I refer to these bids as smart because they recognize the pattern in the bids of different groups and respond accordingly.

¹² In other models (not shown, available upon request), I added an interaction term between the two race-specific memory variables shown in Tables 3, 4 and 5. This interaction was significant in more than half of the results, and indicated that when both groups used race-specific memory the level of inequality is increased by 1 unit. The interaction term, however, was not significant in the supplementary “percent using” memory models (i.e., Tables B1 and B2) that I estimated. The latter insignificant result suggests the possible existence of a non-linear interaction where the magnitude of the interaction increases exponentially—and becomes significant—as we approach the extreme case where all agents use race-specific memory. I exclude the interaction here for simplicity and, more importantly, to avoid making a potentially false inference from a case study at the limits. Readers may interpret the results shown here as a robust, conservative estimate of the impact of race-specific memory on inequality in the Nash bargaining game.

¹³ This table simplifies an extremely complex, nonlinear relationship between the percentage of subordinate agents and level of inequality in the simulations. Specifically, the percentage of subordinate agents uniquely interacts with discrimination, dominant memory and subordinate memory to shape the amount of inequality in a given simulation. The best way to observe these non-linear interactions is by stratifying the regression models shown in Tables 3 and 4 by the percentage in the subordinate population—I did this by dividing the sample into 10 subsamples where each represents a 5 percentage point category in the interval 0 and 50 (e.g., 0-5, 5-10, etc). At the mean (i.e., 25% subordinate), the results mirror those shown here indicating that dominant and subordinate memory jointly drive racial inequality. The stratified results, however, indicate that discrimination and dominant memory play a significantly larger role in the emergence and maintenance of inequality when the percent of subordinate agents is small. The contribution of dominant memory in simulations with 5% subordinates to inequality is roughly negative 10 points. As the percent of subordinate agents in a simulation moves from 25 to 40, though, the impact of subordinate memory dramatically increases to a peak and begins to decline as the percent of subordinate agents in a simulation moves toward 50. The contribution of minority memory to inequality at its

peak is approximately negative 6 points. For brevity and parsimony, I refrain from presenting these more complex results here. They are available by request.

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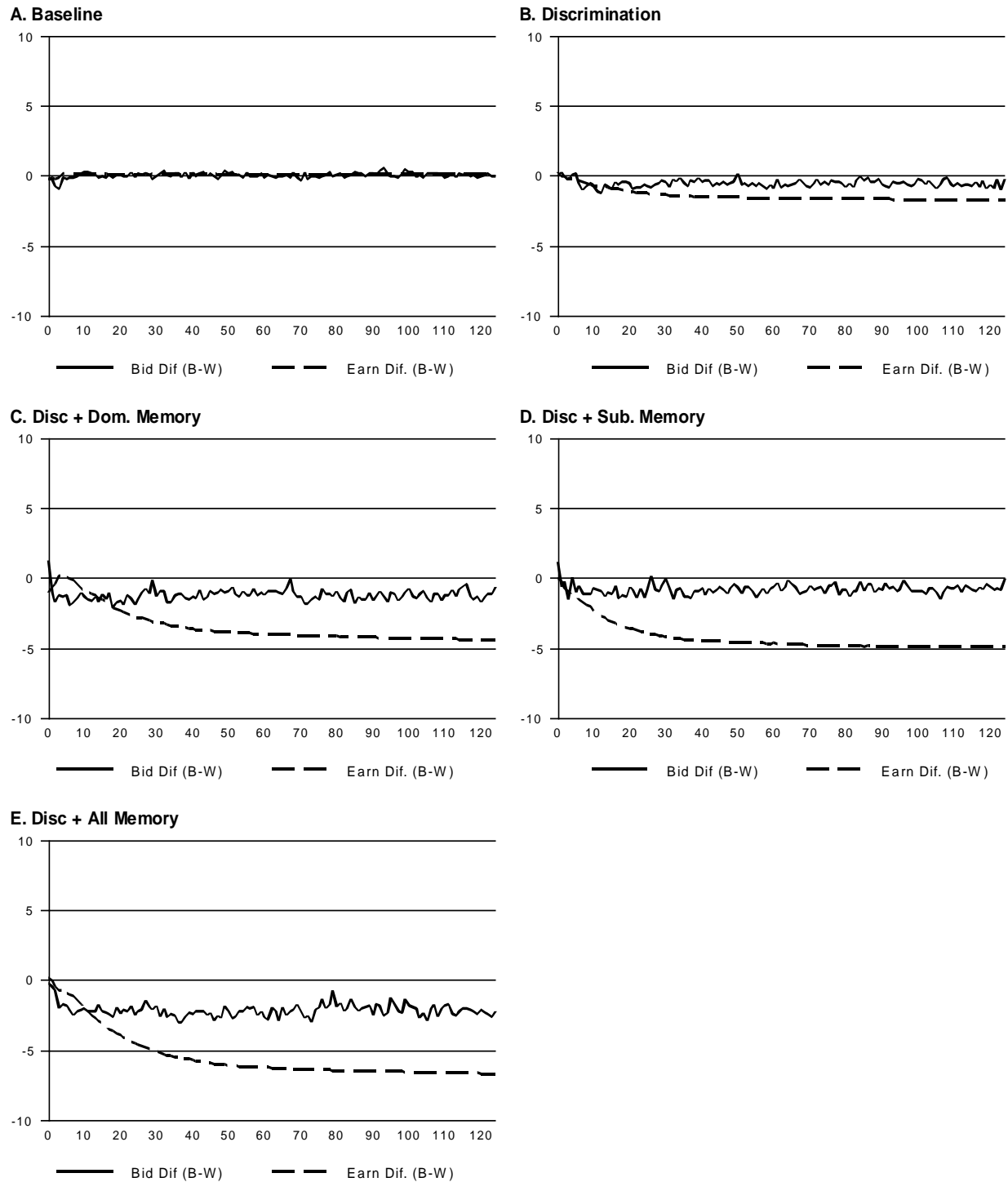
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Figure 1: Group Differences in Average Bids and Earnings over 125 Rounds of Bargaining Game Under Five Specifications



Note: Differences are the average of 50 simulations of the bargaining game with parameters randomly varying for maximum agent age, percent of subordinate agents, memory size, memory noise, bid noise, percent inherited wealth and cost of life. Baseline model (A) has no discrimination or race specific memories—and no measurable group inequality. Discrimination only model (B) has significant difference in favor of dominant group. The other models all have significant differences in earnings and bids.

Table 1: Descriptive Statistics for Simulation Parameters

	<i>Mean</i>	<i>Minimum</i>	<i>Maximum</i>
Agent Life Exp.	29.84	10	50
% Discriminators	12.60	0	25
Magnitude of Disc.	10.64	1	20
Disc. (Mag. x % Disc)	1.34	0	5
% Subord. Agents	25.23	1	50
Memory Size	8.50	2	15
Memory Noise	2.70	1	5
Bid Noise ^a	3.74	1	8
Cost of Life	12.37	0	25
% Wealth Inherited	56.02	10	100
# Rounds	146.46	30	260
# Rounds w/ Disc.	88.55	30	258
# Rounds w/o Disc	57.92	0	228
Avg. Earning Disparity ^b	-4.48	-22.32	6.43
Avg. Wealth Disparity ^b	-262.71	-5135.30	757.25
Recent Earning Disparity ^c	-3.84	-30.52	8.35

Notes: The total number of simulations run (N) was 2,500. All of the parameters shown here—and others—randomly varied across the simulations based on a uniform probability distribution across a pre-specified range.

a. Bid noise is measured as the absolute value of the change in bid per agent in the simulation. It is a product of the prevalence and magnitude of bid noise each run of the simulation.

b. Average group disparities refer to the average group difference in earnings/wealth over the course of all encounters in a bargaining game.

c. Recent earnings disparities refer to the average group difference in earnings over the final 10 encounters of a bargaining game.

Table 2: Multivariate OLS Regression of Average Group Earnings^a Disparities on Simulation Parameters—Games with only Discrimination

	<i>Model 1</i>	<i>Model 2</i>	<i>Model 3</i>
Intercept	-0.4985***	-1.5071***	-0.6087*
Discrimination	-0.6304***	-0.5620***	-0.5668***
# Rounds w/o Disc.	0.0123***	0.0125***	
# Rounds	-0.0063***	-0.0063***	
% Subord.	0.04425**	0.0413**	
% Subord. Squared	0.0002	0.0002	
% Wealth Inherited	-0.0032	-0.0026	
Initial Wealth Disparity	0.1454	0.1194	
Life Expectancy			-0.0253***
Memory Size			-0.0331**
Memory Noise			-0.02405
Bid Noise			0.0533*
Cost of Life			-0.00018
r^2	17.96	27.66	48.98
N	607	607	607
df	1	7	12

Notes: * = $p < 0.05$, ** = $p < 0.01$, *** = $p < 0.001$.

a. Average group earnings disparities refer to the average group difference in earnings over the course of all the encounters in the respective bargaining game.

Table 3: Multivariate OLS Regression of Average Group Earnings^a Disparities on Simulation Parameters—Games with Discrimination & Memory

	<i>Model 1</i>	<i>Model 2</i>	<i>Model 3</i>	<i>Model 4</i>	<i>Model 5</i>	<i>Model 6</i>
Intercept	-3.5324***	-2.9581***	-2.6660***	-0.1962	-4.3656***	-3.8662***
Discrimination	-0.7039***			-0.7092***	-0.6407***	-0.6343***
Dominant Memory		-3.0180***		-3.0299***	-3.1395***	-3.1103***
Subordinate Memory			-3.5368***	-3.5239***	-3.3822***	-3.4035***
# Rounds w/o Disc.					0.0114***	0.0106***
# Rounds					-0.0072***	-0.0072***
% Subord.					0.1772***	0.1791***
% Subord. Squared					-0.0001	-0.00016
% Wealth Inherited					0.0005	0.00178
Initial Wealth Disparity					0.2971	0.24787
Life Expectancy						-0.0474***
Memory Size						-0.00788
Memory Noise						-0.07291
Bid Noise						0.3177***
Cost of Life						-0.00433
r^2	3.06	9.79	13.44	26.29	53.28	57.3
n	2500	2500	2500	2500	2500	2500
df	1	1	1	3	9	14

Notes: * = $p < 0.05$, ** = $p < 0.01$, *** = $p < 0.001$.

a. Average group earnings disparities refer to the average group difference in earnings over the course of all the encounters in the respective bargaining game.

Table 4: Multivariate OLS Regression of Recent Group Earnings^a Disparities on Simulation Parameters—Games with Discrimination & Memory

	<i>Model 1</i>	<i>Model 2</i>	<i>Model 3</i>	<i>Model 4</i>	<i>Model 5</i>	<i>Model 6</i>
Intercept	-3.5535***	-1.9961***	-1.9972***	0.14748	-4.9035***	-4.7822***
Discrimination	-0.2165*			-0.2242**	-0.1636***	-0.1560*
Dominant Memory		-3.6722***		-3.6723***	-3.7726***	-3.7467***
Subordinate Memory			-3.6070***	-3.5990***	-3.4833***	-3.5041***
# Rounds w/o Disc.					0.0041*	0.00339
# Rounds					0.0014	0.00136
% Subord.					0.2095***	0.2118***
% Subord. Squared					-0.0011*	-0.0011**
% Wealth Inherited					0.00153	0.00289
Initial Wealth Disparity					0.25788	0.23039
Life Expectancy						-0.0413***
Memory Size						0.0136
Memory Noise						-0.0607
Bid Noise						0.3152***
Cost of Life						-0.0059
r^2	0.25	12.40	11.92	24.59	43.65	46.63
n	2500	2500	2500	2500	2500	2500
df	1	1	1	3	9	14

Notes: * = $p < 0.05$, ** = $p < 0.01$, *** = $p < 0.001$.

a. Recent group earnings disparities refer to the average group difference in earnings over the final 10 encounters of the bargaining game.

Table 5: Multivariate OLS Regressions of Recent Group Earnings^a Disparities on Simulation Parameters—Stratified into 4 Groupings based on Bargaining Iterations since Discrimination Stopped

	<i>Model A:</i> <i>LT^b 10</i>	<i>Model B:</i> <i>10-to-50</i>	<i>Model C:</i> <i>50-to-100</i>	<i>Model D:</i> <i>GT^b 100</i>
Intercept	-7.6078***	-2.6417***	-4.7504***	-3.7947***
Discrimination	-0.6545***	-0.0605	-0.0855	-0.0140
Dominant Memory	-4.0852***	-3.7303***	-3.4991***	-3.6243***
Minority Memory	-3.7566***	-3.7371***	-3.6009***	-2.9342***
# Rounds w/o Disc.	0.2488***	0.0009	0.0016	0.0090
# Rounds	0.0014	-0.0025	-0.0006	-0.0079
% Subord.	0.3018***	0.0906*	0.2262***	0.2770***
% Subord. Squared	-0.0022*	0.0009	-0.0014	-0.0024**
% Wealth Inherited	0.0125	-0.0010	0.0142**	-0.0101
Initial Wealth Disparity	0.4834	-0.06842	-0.0309	0.9731*
Life Expectancy	-0.0563***	-0.0310***	-0.0308**	-0.0573***
Memory Size	-0.0051	0.0066	-0.0297	0.0838
Memory Noise	-0.0073	-0.0876	-0.0085	-0.0738
Bid Noise	0.4153***	0.3377***	0.2451***	0.2450**
Cost of Life	0.0061	0.0103	-0.0029	-0.0426
r^2	53.83	45.59	49.01	49.78
n	463	891	619	527
df	14	14	14	14

Notes: * = $p < 0.05$, ** = $p < 0.01$, *** = $p < 0.001$. Each of the four models (i.e., A through D) is mutually exclusive, and, as a result, highlights the relationship between the variables under four distinct conditions pertaining to the time since discrimination in the game ceased to exist.

a. Recent group earnings disparities refer to the average group difference in earnings over the final 10 encounters of the bargaining game.

b. The terms LT and GT refer to “Less Than” and “Greater Than,” respectively.

Table B1: Multivariate OLS Regression of Average Group Earnings^a Disparities on Simulation Parameters—Games with Discrimination & “Percent” using Memory Variable

	<i>Model 1</i>	<i>Model 2</i>	<i>Model 3</i>	<i>Model 4</i>	<i>Model 5</i>	<i>Model 6</i>
Intercept	-3.9805***	-3.2597***	-3.2933***	-0.9665***	-5.9132***	-5.8336***
Discrimination	-0.5988***			-0.6186***	-0.5904***	-0.5748***
% Dominant Memory		-3.1231***		-3.1704***	-3.3552***	-3.2553***
% Subordinate Memory			-3.0101***	-2.9572***	-3.1891***	-3.2302***
# Rounds w/o Disc.					0.0095***	0.0091***
# Rounds					-0.0067***	-0.0064***
% Subord.					0.2534***	0.2478***
% Subord. Sq.					-0.0010***	-0.0009***
% Wealth Inherited					-0.0009	-0.0008
Initial Wealth Disparity					0.1032	0.1144
Life Expectancy						-0.0468***
Memory Size						0.0451***
Memory Noise						-0.1133**
Bid Noise						0.3336***
Cost of Life						-0.0006
r^2	2.95	4.73	4.48	12.25	61.98	67.80
n	2500	2500	2500	2500	2500	2500
df	1	1	1	3	9	14

Notes: * = $p < 0.05$, ** = $p < 0.01$, *** = $p < 0.001$.

a. Average group earnings disparities refer to the average group difference in earnings over the course of all the encounters in the respective bargaining game.

Table B2: Multivariate OLS Regression of Recent Group Earnings^a Disparities on Simulation Parameters—Games with Discrimination & “Percent” Using Memory Variables

	<i>Model 1</i>	<i>Model 2</i>	<i>Model 3</i>	<i>Model 4</i>	<i>Model 5</i>	<i>Model 6</i>
Intercept	-4.0380***	-2.3697***	-2.6875***	-0.6446***	-6.6705***	-6.8646***
Discrimination	-0.1409***			-0.1654***	-0.1480**	-0.1338**
% Dominant Memory		-3.8297***		-3.8159***	-4.0442***	-3.9404***
% Subordinate Memory			-3.1280***	-3.0802***	-3.3353***	-3.3809***
# Rounds w/o Disc.					0.0034*	0.0030
# Rounds					0.0012	0.0015
% Subord.					0.3001***	0.2952***
% Subord. Sq.					-0.0021***	-0.0020***
% Wealth Inherited					0.0004	0.0004
Initial Wealth Disparity					-0.0243	0.0070
Life Expectancy						-0.0398***
Memory Size						0.0859***
Memory Noise						-0.1397**
Bid Noise						0.3208***
Cost of Life						-0.0149*
r^2	0.14	6.04	4.10	10.22	49.67	54.23
n	2500	2500	2500	2500	2500	2500
df	1	1	1	3	9	14

Notes: * = $p < 0.05$, ** = $p < 0.01$, *** = $p < 0.001$.

a. Recent group earnings disparities refer to the average group difference in earnings over the final 10 encounters of the bargaining game.