Modeling Mortality Surfaces: The Influence of Period and Cohort Components on Elderly Mortality

Elisabetta Barbi[1,2] and Carlo G. Camarda[2]

[1] *Sapienza University of Rome, Italy*

[2] *Max Planck Institute for Demographic Research, Rostock - Germany*

Extended abstract for the PAA annual meeting
Dallas, 15-17 April 2010

1 Introduction

Mortality surfaces are a powerful tool for demographic research. The impact of age, period, and cohort effects on mortality is easily traced in the graphic representation of mortality rates represented as function of age and time. At aggregate level, hence, the analysis of mortality surfaces offers the best opportunity to address one of the topics that is increasingly becoming of great interest for demographers: the relative importance of cohort versus period effects on elderly mortality. What are the main factors affecting adult and old mortality? How much continuity is there in an individual's life and how much plasticity? A large body of studies addresses the question about the importance of early-life and current conditions for mortality in later-life, including studies that account for a quite wide range of hypotheses with possibly different implications not only for demographers when forecasting mortality but also for policy makers when planning public-health interventions (for reviews see Elo and Prestom 1992, Doblhammer 2004, Myrskyla, 2009).

In this paper we consider two main questions: To what extent do early-life and current factors determine trends in old-age mortality? Did early life factors determine the decline of adult and old mortality in the second half of the 20^{th} century in the European countries? We present a new explanatory but flexible relational model to analyse mortality surfaces including period and cohort indicators. This two-dimensional model is estimated via a semi-parametric approach by smoothing the general mortality trend over age and time with penalized B-splines. This allows, on one hand, to keep the non-linear structure of mortality dynamics over the long term, and on the other hand to evaluate simultaneously the relative importance of period and cohort effects. We apply the described model to two case studies, Sweden and Italy, which have been chosen because of their different past mortality dynamics.

2 Smoothing mortality surfaces

Graphic representations of mortality surfaces as Lexis maps over age and time are a powerful tool for a first inspection of age, period and cohort effects and for descriptive purposes. Detailed analyses, however, demand analytical tools that can extract structural information from surfaces of mortality rates more accurately. Mortality surfaces may be analyzed through various parametric specifications (Lee Carter, 1992; Vaupel, 1998) that may be seen as part of the general class of relational models and estimated using the maximum likelihood method. Also the model presented here may be considered as a relational model.

Let

$$\bar{\mu}(x, y) = \mu_0(x, y) \cdot \exp\left(\alpha \cdot I_p + \beta \cdot I_c\right) \tag{1}$$

be the force of mortality at age $x, x = 1, \ldots, m$ and time $y, y = 1, \ldots, n$, where I_p and I_c are two indicators of period and cohort factors respectively, α and β measure the impact of these period and cohort indicators respectively, and $\mu_0(x, y)$ is a standard mortality surface, representing the general mortality trend. This two-dimensional baseline may be obtained through empirical observations or parametric functions. Both the period and cohort factors have been synthesized through the evolution of infant mortality levels during the years (n) and the cohorts $(m + n - 1)$ involved in the study period. Thus the model may be written in the following form:

$$\bar{\mu}(x, y) = \mu_0(x, y) \cdot \exp\left(\alpha \cdot \frac{\mu(0, y) - \mu^n_{\min}(0)}{\mu^n_{\max}(0) - \mu^n_{\min}(0)} + \beta \cdot \frac{\mu(0, y - x) - \mu^{m+n-1}_{\min}(0)}{\mu^{m+n-1}_{\max}(0) - \mu^{m+n-1}_{\min}(0)}\right) \tag{2}$$

where

$$\mu^k_{\min}(0) = \min_{y=1\ldots k} \mu(0, y)$$
$$\mu^k_{\max}(0) = \max_{y=1\ldots k} \mu(0, y)$$

However this choice may be not optimal, especially for representing period factors. Other synthetic indicators such as improvements in the cumulative probability function or shifts in the modal age at death may be more suitable.

The inclusion of various components in fully parametric models can be carried out in different ways. However, identifiability and computational issues may arise, leading to unstable maximum likelihood estimates. For this reason, we follow a semi-parametric approach, by smoothing the standard mortality surface over age and time. Assuming that death counts are a realization of a Poisson distribution, we can use two-dimensional regression splines, specifically B-splines with penalties, known as P-splines, to describe the baseline $\mu_0(x, y)$ (Eilers and Marx, 1996; Currie, Durban and Eilers, 2004). In this way, we let the data speak by themselves without influencing the estimation of α and β.

3 Applications to Sweden and Italy

Our analyses focus on adult and old-age mortality (ages 50-99) in the second half of the 20^{th} century in two countries with different mortality experiences, Sweden and Italy, for men and women separately. These two countries differ in the way and speed in which they experienced the demographic transition. Like other Northern European countries, Sweden is characterized by an early start of the mortality transition. In contrast Italy, at least until the beginning of the 20^{th} century, experienced a delay with respect to the Northern European countries, in particular with respect to Sweden, Norway and England. Italian infant mortality reached the Swedish level only around 1990 but Italian death rates at adult-old ages fell to Swedish levels about four decades earlier. Furthermore, while Italy participated in World War I and World War II, Sweden was neutral during both world wars.

For our purpose of modelling mortality surfaces and determining the impact of current conditions and that of early-life factors on adult and elderly mortality, data were needed not only for Italian and Swedish men and women aged 50-99 from 1950 onwards, but also for all the cohorts involved in the study period. More precisely, we analysed Swedish data from 1950 to 2006, accounting for mortality data of the cohorts born from 1851 to 1956, and Italian data from 1971 to 2006, accounting for mortality data of the cohorts born from 1872 to 1956. These data were obtained from the Human Mortality Database (www.mortality.org).

With their different mortality profiles, these two countries certainly are good candidates to test a statistical model which aims at the decomposition of mortality surfaces into period and cohort mortality patterns.

References

Barbi E., Vaupel J.(2005), Comment on Inflammatory Exposure and Historical Changes in Human Life-Spans, *Science*, vol. 308.

Currie I.D., Durban M. and Eilers P.H.C.(2004), Smoothing and forecasting mortality rates, *Statistical Modelling*, 4. 279-298

Doblhammer G. (2004), *The late life legacy of very early life.* Demographic Research Monographs, Springer Verlag, Heidelberg.

Eilers PH, Marx BD (1996), Flexible smoothing with B-splines and penalties, *Statistical Science* 11(2).

Finch C., Crimmins E. (2004), Inflammatory Exposure and Historical Changes in Human Life-Spans, *Science*, vol. 305.

Lee, R.D. and L.D. Carter (1992). Modeling and Forecasting U.S. Mortality. *Journal of the American Statistical Association*, 87: 659-671.

Vaupel JW (1999), The long-term pattern of adult mortality and the highest attained age. Discussion on the paper by A R Thatcher. *Journal of the Royal Statistical Society* - Series A: Statistics in Society. 162: 31-32.

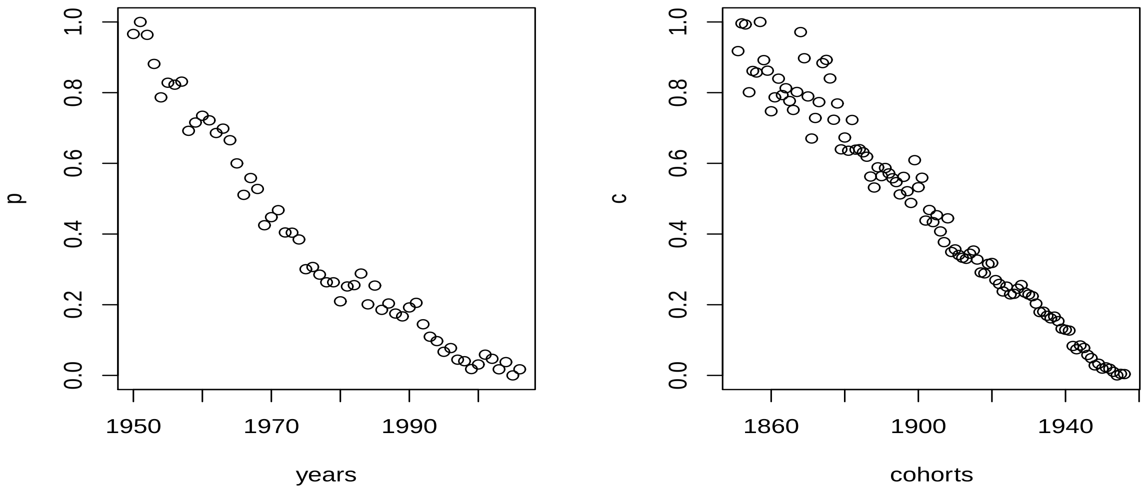

Figure 1: Period and cohort infant mortality (standardized). Sweden, women.

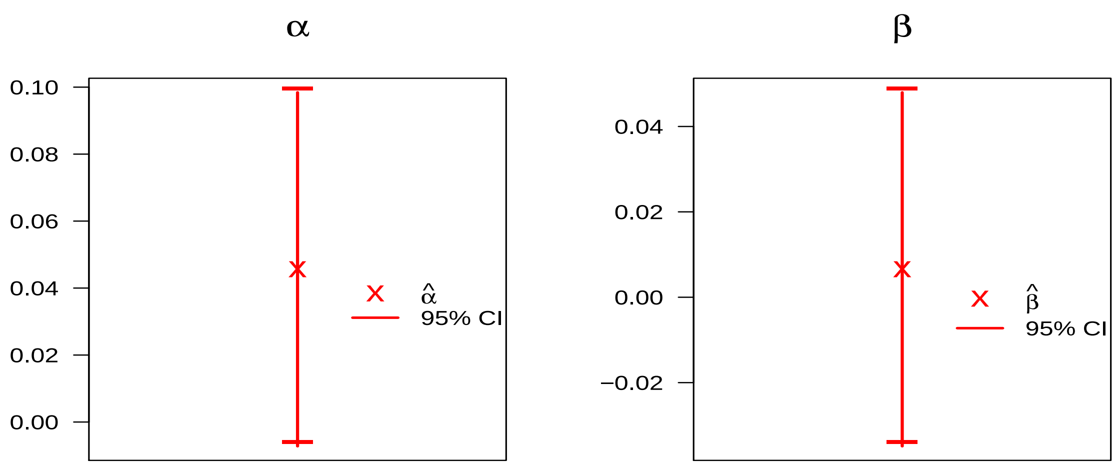

Figure 2: Estimates and 95% confidence interval for α and β. Sweden, women.

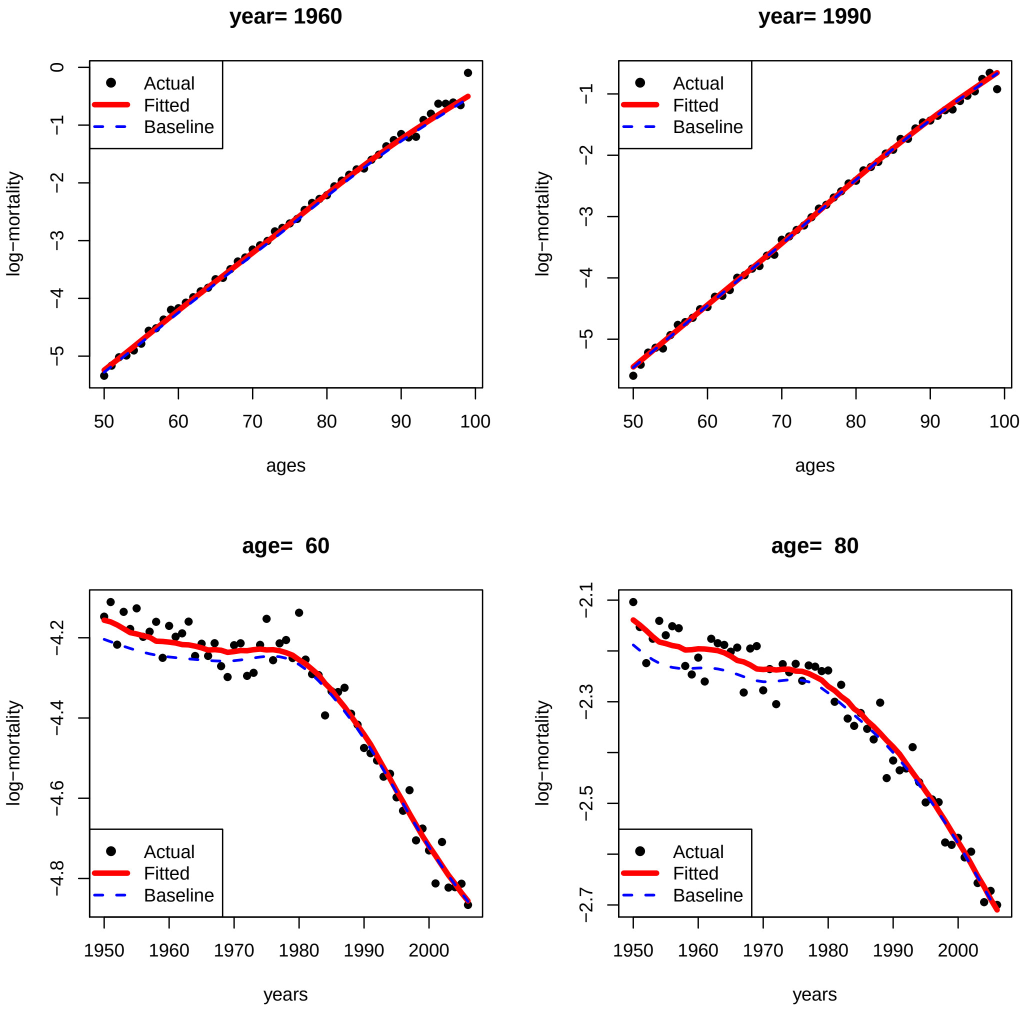

Figure 3: Observed and fitted mortality rates for selected years and ages. Sweden, women.

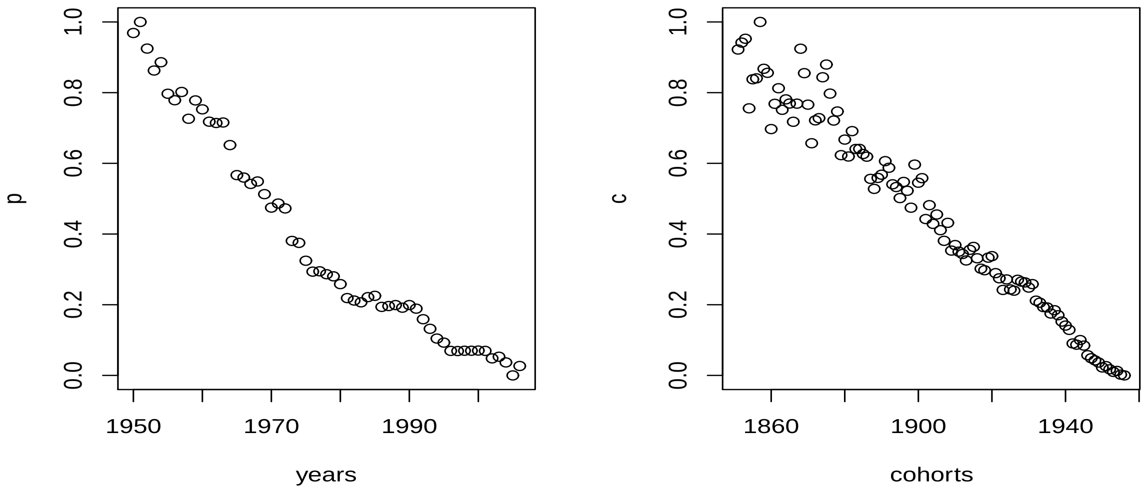

Figure 4: Period and cohort infant mortality (standardized). Sweden, men.

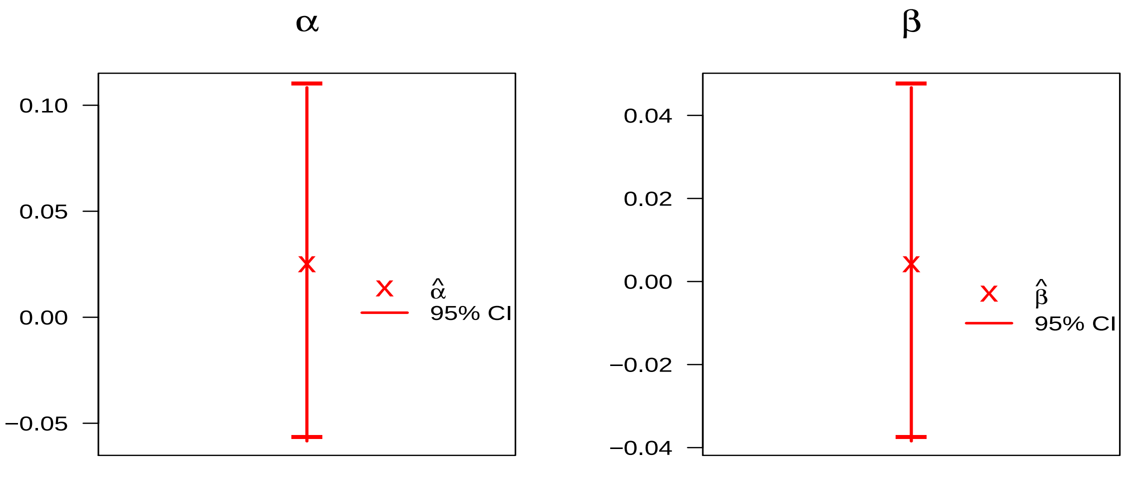

Figure 5: Estimates and 95% confidence interval for α and β. Sweden, men.

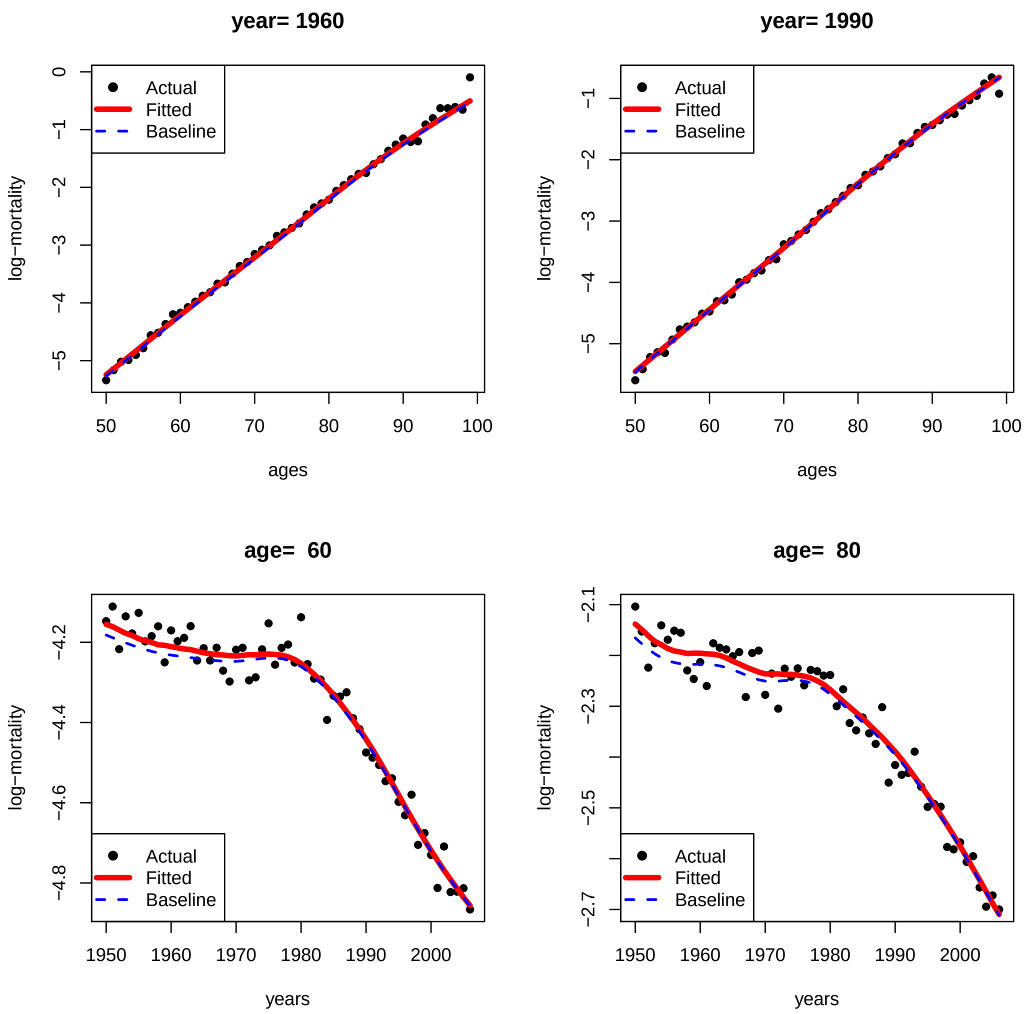

Figure 6: Observed and fitted mortality rates for selected years and ages. Sweden, men.

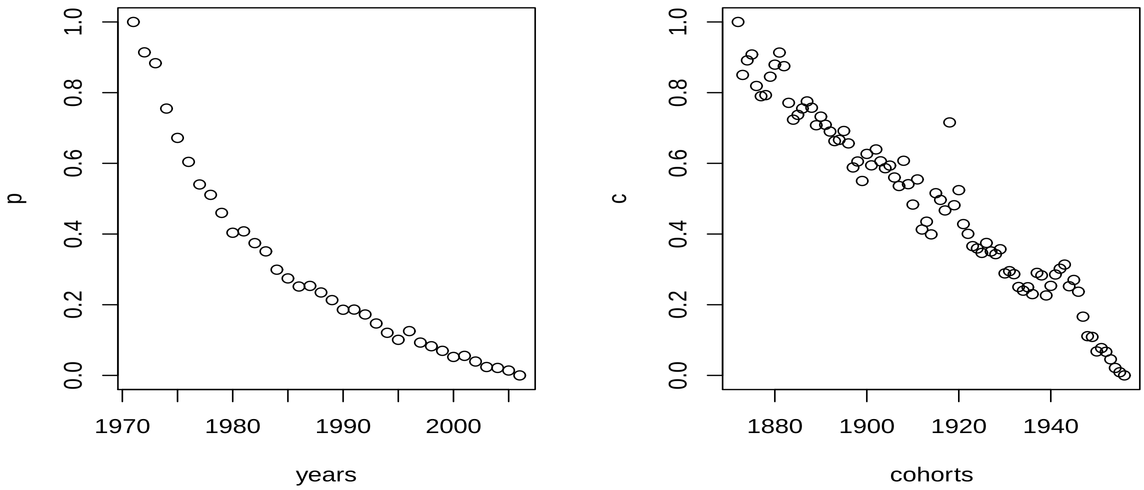

Figure 7: Period and cohort infant mortality (standardized). Italy, women.

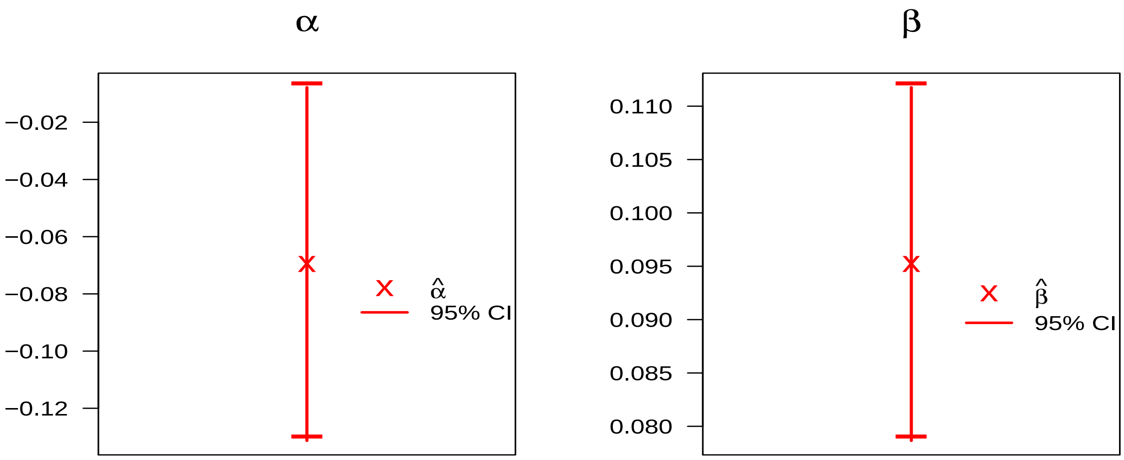

Figure 8: Estimates and 95% confidence interval for α and β. Italy, women.

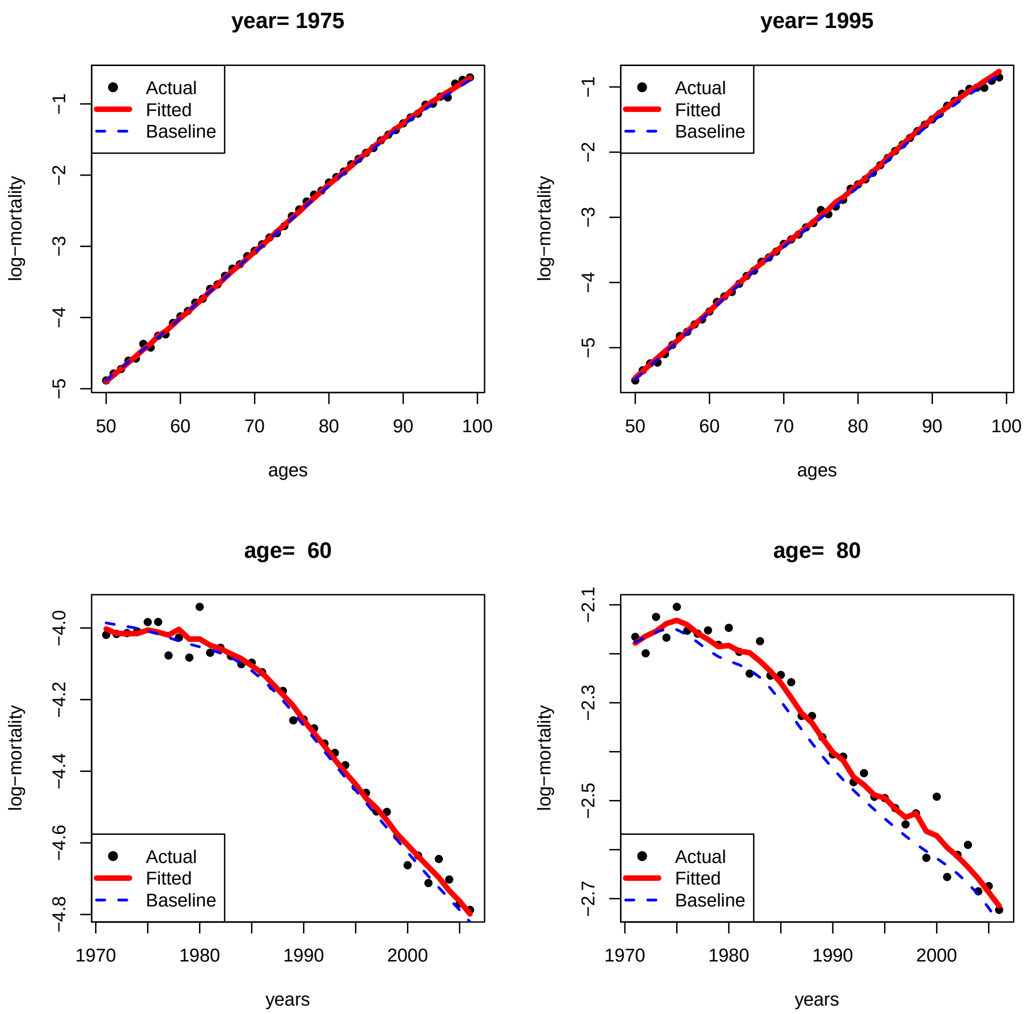

Figure 9: Observed and fitted mortality rates for selected years and ages. Italy, women.

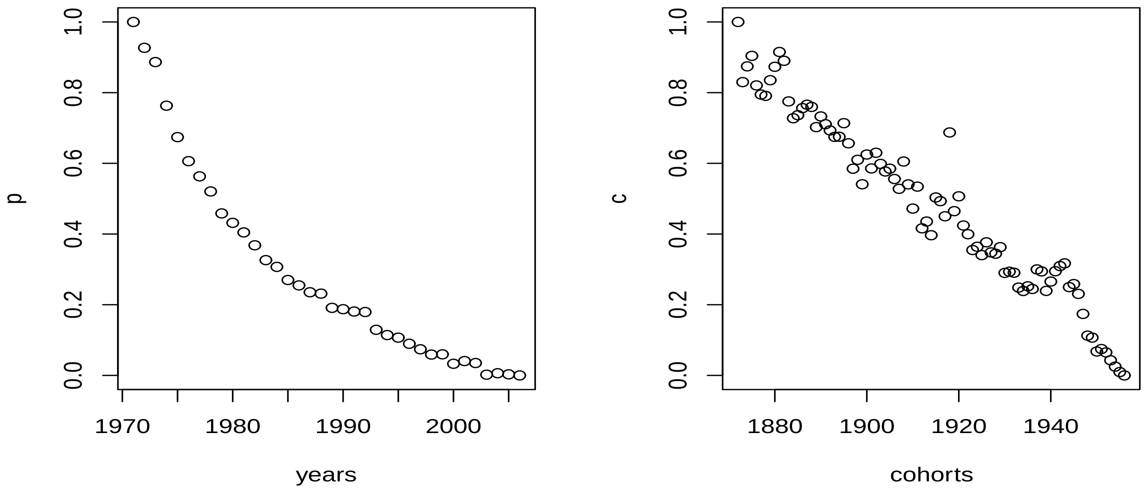

Figure 10: Period and cohort infant mortality (standardized). Italy, men.

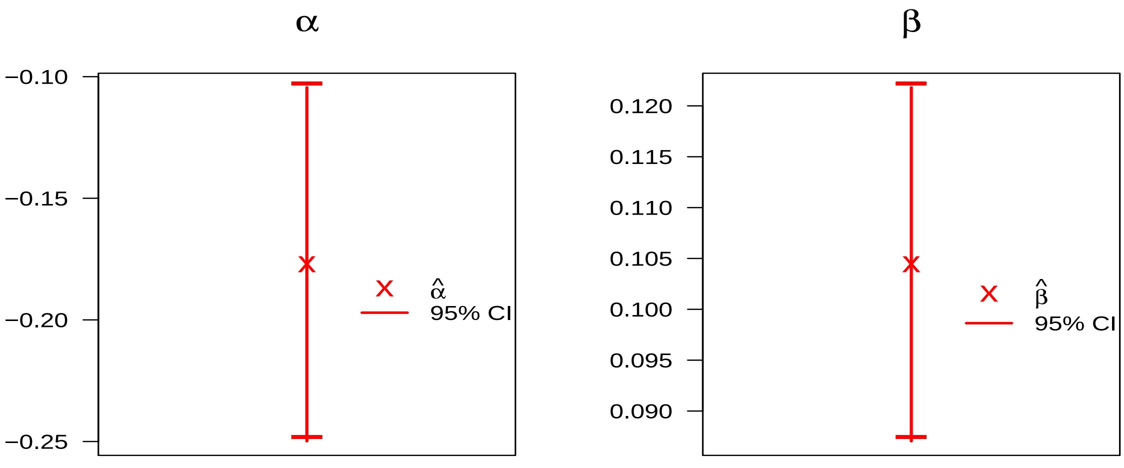

Figure 11: Estimates and 95% confidence interval for α and β. Italy, men.

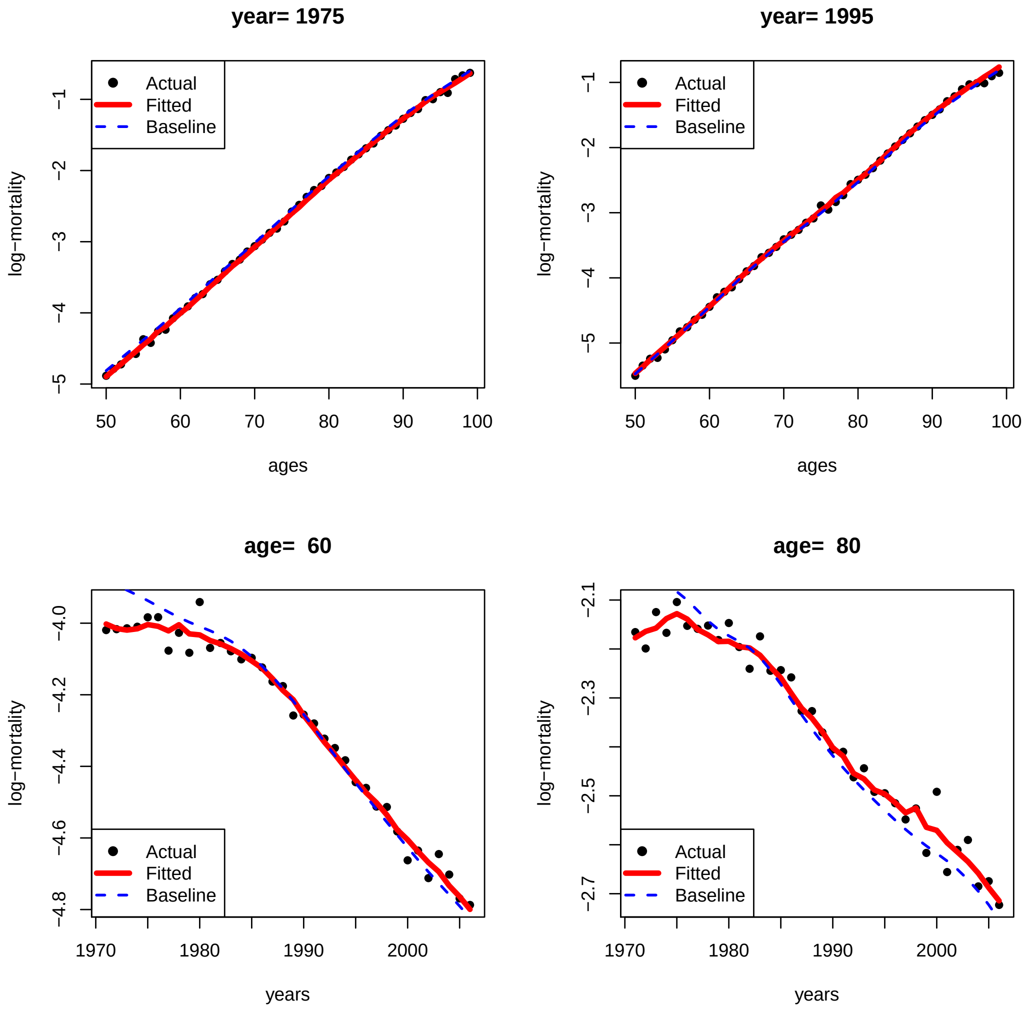

Figure 12: Observed and fitted mortality rates for selected years and ages. Italy, men.