On the correspondence between CAL and lagged cohort life expectancy

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Abstract

It has been established that under the linear shift assumption, CAL(t) is equal to the life expectancy for the cohort born at time t-CAL(t), or, equivalently, $e_0^c(t)$ is equal to CAL for the period t+ $e_0^c(t)$. This correspondence is important, because the cohort life expectancy for the cohort currently reaching its life expectancy, or lagged cohort life expectancy (LCLE), has been discussed in the tempo literature as a summary mortality measure of substantive interest. In this paper, we establish that the CAL-LCLE correspondence holds in a variety of empirical situations, present or historical, including ones in which the linear shift assumption doesn't apply, and we provide some more general principles about the extent to which CAL can be used as an estimate of LCLE. Finally, we discuss the implications of the CAL-LCLE correspondence for using CAL (or LCLE) as a summary mortality measure, and for the projection of cohort mortality.

Introduction

In recent years, several alternative summary measures of mortality have emerged in the context of the debate on mortality tempo. The existence of tempo effects in mortality is still a controversial issue. Nonetheless, this debate has generated a healthy discussion about various ways to summarize a population's mortality experience.

One measure that has been discussed in the context of this debate is CAL, the cross-sectional average length of life. CAL's original purpose was not to address tempo effects in mortality. In fact, CAL was developed prior to Bongaarts and Feeney's proposition that there are tempo effects in mortality, and it has its own interpretation aside from tempo effects. Nonetheless, CAL is closely related to Bongaarts and Feeney's tempo adjusted life expectancy. In particular, the two indicators are equal to one another when mortality follows some specific patterns.

One other finding that has emerged from the tempo debate is the fact that, under some assumptions, CAL is equal to the value of the cohort life expectancy for the cohort currently reaching this value. The life expectancy for the cohort currently reaching its life expectancy is an interesting index in itself. However, it cannot be calculated for the current year, because in a cohort many individuals live beyond their cohort's life expectancy or mean age at death. Therefore a cohort's life expectancy or mean age at death is not known until the cohort is extinct, i.e., many years after the time at which that mean age at death is reached. CAL, however, can be readily calculated for the current year with no reference to the future, as long as sufficient historical mortality data is available. Therefore, the existence of a correspondence between CAL and LCLE is of significant interest for two main reasons. First, it could potentially provide CAL with an additional, concrete and useful interpretation. Second, it could provide CAL with a use in the area of mortality projections, and help estimate life expectancy for cohorts that are not yet extinct.

In this paper, after defining these two measures, we examine the correspondence between CAL and LCLE in actual populations. We then examine the mortality assumptions under which the correspondence holds, exactly or approximately. Finally, we use simulations to examine the extent to which the correspondence holds under more general mortality scenarios.

I. CAL and LCLE

a) CAL, the Cross-sectional Average length of Life

CAL is defined as follows:

$$CAL(t) = \int_0^\infty p_c(x, t-x)dx \qquad \textbf{(1)}$$

where $p_c(x, t-x)$ is the probability of surviving from birth to age x for the cohort born at time t-x. In a population, $p_c(x, t-x)$ corresponds to the proportion of cohort survivors for the cohort aged x at time t. Simply put, CAL is the cross-sectional sum of proportions of cohort survivors at a given time. CAL is a mortality measure that summarizes the mortality history of all cohorts present in a population at a given time. It also corresponds to the size of the "birth-standardized", or "constant-births" population, i.e., the total size of a population with a constant stream of births exposed to actual, changing mortality trends.

Equation (1) takes into account mortality at all ages. Like the life expectancy, CAL can also be calculated for any starting age x:

$$CAL_x(t) = \int_x^{\infty} \frac{p_c(a,t-a)}{p_c(x,t-a)} da \qquad \textbf{(2)}$$

By involving cohort survival to age *a* conditional on survival to age *x*, CAL_x takes into account mortality rates above age x only.

CAL was first defined by Brouard (1986). Further developments where introduced by Guillot (1999, 2003), including equations for the relationship between CAL change and period mortality, and the population size interpretation of CAL. Guillot (2005) has also shown that the discrepancy between e_0 and CAL during a given year can be interpreted in terms of population momentum. More recently, Wachter (2005) has shown that CAL is essentially a weighted average of past levels of period life expectancy.

In a series of articles, Bongaarts and Feeney have proposed two alternative summary mortality measures, which are closely related to CAL: M_2, also referred to as MAD; and M_4 (Bongaarts and Feeney, 2003). If mortality follows a specific pattern of change referred to as the "proportionality assumption", CAL, MAD and M_4 are equal to one another (Bongaarts and Feeney 2003).

b) LCLE, the lagged cohort life expectancy

Each cohort that is now extinct has a known value for its life expectancy at birth:

$$e_0^C(t) = \int_0^{\infty} p_c(x,t)dx \qquad \textbf{(3)}$$

In closed populations, cohort life expectancy is equal to the observed mean age at death for that cohort. For cohorts that are not yet extinct, their remaining mortality experience is unknown, and thus their life expectancy or mean age at death is unknown. Unless some assumption about future mortality is made, cohort life expectancy can be calculated with certainty only for cohorts that are extinct or near extinct.

The year at which a cohort born in year *c* reaches its life expectancy is $t = c + e_0^c(c)$. Year *t* is an important year for cohort *c*; it is the mean year of death for that cohort. If there was no variation in ages at death, year *t* would be the year in which all members of cohort *c* die.

Lagged cohort life expectancy, or LCLE, is simply a graphical representation of the classic cohort life expectancy. Instead of plotting cohort life expectancy against the cohort's year of birth *c*, as commonly done, cohort life expectancy is plotted against its mean year of death, or

$c+e_0^c(c)$. This lag provides a useful time reference for an indicator that summarizes a mortality experience spread over many years, but centered around its mean year of death. This graphical representation of cohort life expectancy is somewhat similar to the well-known representation of the cohort TFR, often plotted against the time at which a cohort reached its mean age at birth.

LCLE(t) can be interpreted as the life expectancy for the cohort reaching its life expectancy during year t. Evidently it is not possible to know with certainty which cohort is currently reaching its life expectancy, because that cohort has not yet completed its mortality trajectory. LCLE can only be calculated only for past years, using retrospective mortality information for cohorts that are now extinct. However, even if LCLE cannot be calculated for the current year, the concept it represents is useful.

There can be more than one cohort currently reaching its life expectancy. This happens when a cohort has a life expectancy that is at least one year below that of the preceding cohort. While large annual changes in period life expectancy are not uncommon, cohort life expectancy changes much more gradually. In the human mortality database that we use in this paper, there are only a few exceptional cases in which e_0^c changes by one year or more for two consecutive years. For the overwhelming majority of years, there is only one value of LCLE(t) associated with each year t.

c) Correspondence between the two indexes

The parallel between CAL and the life expectancy of an actual cohort was first drawn by Guillot (2003, pp. 47-48). However, the observation that the current CAL value might correspond to the cohort life expectancy of the cohort born CAL years earlier, or that a cohort's life expectancy might correspond to the CAL value observed at the time when that cohort reaches its life expectancy, emerged as part of the debate on tempo effects. Bongaarts and Feeney (2006) observed that in Denmark, England & Wales, and Sweden, the lagged cohort mean age at death is relatively close to MAD(t) when there is no mortality below age 30. Rodriguez (2006) and Goldstein (2006) show that when cohort survival shifts linearly overtime (an assumption that we describe later), there is perfect correspondence between CAL and LCLE. Finally, Bongaarts and Feeney (2005) simulate a Gompertz model with a constant rate of improvement over time, and find a near-exact correspondence between CAL, LCLE and M_4 over the 50-year period of their simulation.

Bongaarts and Feeney view the correspondence between CAL, LCLE and M_4 as evidence that their tempo-adjusted measure adjusts of tempo bias. The logic behind this conclusion is that cohort life expectancy is a mortality indicator free of tempo bias, and that the cohort life expectancy for the cohort currently reaching its life expectancy is an appropriate value against which period life expectancy can be compared to assess the scale of tempo bias. We will discuss the relevance of this comparison later in the paper.

Regardless of the debate on tempo effects, we believe that studying the correspondence between CAL and LCLE is useful for the reasons we stated earlier. First, if the correspondence holds, this means that CAL can be interpreted as an indicator of the mean age at death for the cohort currently reaching this age, an indicator that is interesting in its own right. Second, if the correspondence holds, this implies that CAL can be used as a device for projecting cohort life expectancy for cohorts that are not yet extinct.

In this paper, we focus on the comparison between LCLE and CAL rather than MAD or M_4. This choice is justified by the fact that CAL and LCLE are both based on the same basic information – cohort survival probabilities, or, equivalently, cohort person-years lived – summed in two different ways: cross-sectionally for CAL; and longitudinally for LCLE. CAL is thus more amenable to mathematical manipulation when comparing it to LCLE, and a more logical choice in empirical comparisons. However, given the similarity between CAL, MAD and M_4, the regularities observed in this paper are likely to also apply to MAD and M_4.

II. Correspondence between CAL and LCLE in actual populations

a) Database

The data that we use in this paper comes from the human mortality database (www.mortality.org). The number of country-years for which we can study the correspondence varies depending on the amount of historical data available and the choice of starting age for the calculation of life expectancies and CAL. In this paper, we choose three starting ages: 0; 30; and 60. The number of country-years increases dramatically when using a starting age of 60.

b) Measurement issues

There are two ways of assessing the correspondence between CAL and LCLE in actual populations. The first option involves the comparison of $e_0^c(t)$ vs. $CAL(t+e_0^c(t))$ for each cohort born in year t. In a graph of CAL vs. LCLE, this difference amounts to the vertical difference between the two indicators. The second option involves comparing CAL(t) vs. $e_0^c(t-CAL(t))$ for each year *t*. In a graph of CAL vs. LCLE, this difference amounts to the vertical projection of the diagonal difference between the two indicators. These two ways of representing discrepancies between CAL and LCLE can each be analyzed by period or by cohort. Thus there are four different ways of representing the correspondence between CAL and LCLE.

When the correspondence exactly holds, these various ways of representing the correspondence provide the same answer. When there is some discrepancy, however, the amount of discrepancy varies slightly depending on the choice of approach. Nonetheless, the results are not substantially different, especially when CAL and cohort life expectancy are calculated on an annual basis.

One complication is that there might be more than one cohort reaching its life expectancy in a given year, as we said earlier. Therefore there could be more than one vertical difference for a given year. Similarly, there could be more than one diagonal difference for a given cohort. Therefore, it would be preferable to study diagonal differences by year or vertical differences by cohort. Given that one of the goals of this paper is to assess how CAL can be used to assess the life expectancy for the cohort currently reaching its life expectancy, we decided to study vertical differences by year. The existence of multiple vertical differences is rare enough that it can be disregarded in the empirical analysis of the mortality database.

Using age-specific death rates for each year and single-year age groups organized by cohorts, we calculated cohort survival probabilities for each annual cohort centered on January 1. These survival probabilities are the used to calculate cohort life expectancy for each cohort centered on January 1. These cohort survival probabilities are then summed cross-sectionally to calculate CAL for each year centered on January 1. The CAL series is then linearly interpolated to calculate the CAL value corresponding to each value of $t+e_0^c(t)$. The difference – absolute or relative -- between $CAL(t+e_0^c(t))$ and $e_0^c(t)$ can then be examined for each year at which $CAL(t+e_0^c(t))$ can be calculated.

When using a starting age other than 0, the CAL-LCLE correspondence is analyzed in a similar fashion. Defining *t* as the year at which a given cohort reaches age *x*, we examine the correspondence between $e_x^c(t)$ and $CAL_x(t+e_x^c(t))$.

c) Example: Swedish females

Figure 1 shows trends in e_0^p, CAL_0 and $LCLE_0$ among Swedish females. The bottom panel presents the absolute difference between CAL_0 and $LCLE_0$ for the years when the two indicators overlap. During these years, the difference evolves from about -4 years to about +4 years.

Figure 2 shows the correspondence when considering mortality above age 30 only. The correspondence dramatically improves in this case. In the later part of the period, however, the difference between CAL and LCLE increases to about 1 year.

When mortality above age 60 only is involved (Figure 3), the correspondence is excellent. The difference is clearly centered around zero, with only very small deviations. There is no particular trend in the amount of difference.

The amount of discrepancy between the two indicators improves between Figure 1 and 3 partly because the level of life expectancy at age 60 is by nature lower than the level of life expectancy at birth. To control for these scale difference, we calculated the relative difference (CAL/LCLE) between the two indicators (Figure 4). There is a clear improvement in the relative difference between CAL and LCLE as the starting age increases. For life expectancy at age 60, the CAL-LCLE correspondence remains excellent.

d) Generalization

Figure 5, 6 and 7 generalize the results of Figure 4 to all countries in the mortality database. The patterns found among Swedish females apply here as well. These figures show that the correspondence between CAL and LCLE is not precise when looking at life expectancy at birth (Figure 5). However, the correspondence is very much improved when we use a starting age of 30 (Figure 6), and it is excellent with a starting age of 60 (Figure 7). For the country-years presented in the Figure 7, we find (retrospectively) that CAL_{60} could have been used as a precise estimate of cohort life expectancy at age 60 for the cohort reaching e_{60} in a given year, even though that that cohort was not yet extinct.

III. Mortality patterns producing produce an exact or near exact correspondence

We have shown the extent to which the correspondence holds in actual populations. In order to gain insight into how generally the correspondence holds, we examine the mortality patterns producing an exact or near exact correspondence.

a) Linear shift pattern of mortality change

The linear shift pattern of mortality change is a pattern in which a baseline schedule of period mortality rates, μ(x, 0), is shifted every year along the age axis by a quantity r:

$$\mu(x,t) = \mu(x - rt, 0) \quad \textbf{(4)}$$

The linear shift assumption also assumes that μ(x,t)=0 for x<rt.

As a consequence of this shifting pattern in period mortality rates, the amount of shift for each successive cohorts will be r/(1-r). Starting from a baseline schedule of cohort mortality rates, $\mu^c(x, 0)$:

$$\mu^c(x,t) = \mu^C(x - \frac{r}{1-r}t, 0) \quad \textbf{(5)}$$

and $\mu^c(x,t)=0$ for x<r/(1-r).

Figure 8 illustrates the linear shift pattern of mortality change and the correspondence between period and cohort shifts. It is important to note here that the linear shift assumption does not make any particular parametric assumption about the mortality schedule.

Rodriguez (2006, pp. 99-100) has shown that, with a linear shift for periods starting at time t=0, CAL will follow a linear trajectory:

$$CAL(t) = e_0^P(0) + rt \quad \textbf{(6)}$$

where $e_0^P(0)$ is the baseline life expectancy embodied in μ(x,0). He also showed that, given a linear shift starting at time t=0, the cohort life expectancy at birth will also change linearly, but at a different rate $r^c = r/(1-r)$:

$$e_0^C(t) = \frac{e_0^P(0)}{1-r} + \frac{r}{1-r}t \quad \textbf{(7)}$$

Combining Equations (5) and (6), Rodriguez (2006) finds that:

$$CAL(t+e_0^C(t)) = e_0^C(t) \quad \textbf{(8)}$$

Or, alternatively, that:

$$e_0^C(t-CAL(t)) = CAL(t) \quad \textbf{(9)}$$

In other words, under the linear shift assumption, the correspondence is perfect. Goldstein (2006) reaches the same conclusion with a somewhat different demonstration. Note that this result is exact and does not involve any other assumptions besides the linear shift assumption described in Equation (3). Again, no assumption is made about the shape of the mortality schedule.

We would like to note here that the linear shift assumption described here is a special case of Bongaarts and Feeney's "proportionality assumption":

$$\mu(x,t) = \mu(x-r(t),0) \quad \textbf{(10)}$$

in which r(t) = rt. While the linear shift assumption produces an exact correspondence, the proportionality assumption per se does not guarantee that CAL and LCLE will agree. For example, in Bongaarts and Feeney's well-known "pill" scenario (i.e., a one-time shift in a cohort mortality happening for all cohorts at the same time), the proportionality assumption is met, but the linear shift assumption is not. In this scenario, CAL increases abruptly to the ultimate level of cohort life expectancy, while e_0^c (and LCLE) adjusts gradually. There is no perfect CAL-LCLE correspondence in this scenario. We will discuss the implications of this scenario later in the paper.

b) Gompertz pattern of mortality with constant log-linear decline

This is a simple model of mortality change in which age-specific mortality follows a Gompertz model with a constant rate of improvement over time:

$$\mu(x,t) = \alpha e^{\beta x} e^{-\rho t} \quad \textbf{(11)}$$

Using simulations, Bongaarts and Feeney (2005) show that when mortality follows such a pattern, CAL is approximately equal to LCLE, i.e., the correspondence holds approximately.

This model of mortality change is in fact quite similar to the linear shift assumption described above. Under this model, as in the linear shift model,

$$\mu(x,t) = \mu(x-rt,0) \quad \textbf{(12)}$$

where r= ρ/β.

One difference, however, with the linear shift described above is that mortality at ages $x<rt$ is not equal to 0. There is some mortality at these ages, following Equation 11. As a result, CAL does not increase exactly by an amount of ρ/β, but by a slightly smaller amount. Similarly, e_0^c does not increase exactly by an amount of $\rho/(\beta-\rho)$, as one would expect under the linear shift assumption. Nonetheless, the additional mortality existing under this scenario is so small that Equations (5) and (6) can be approximated using $r= \rho/\beta$. (See Goldstein and Wachter (2006) for the details of this approximation.) Thus this Gompertz model will produce an approximate correspondence between CAL and LCLE, as Bongaarts and Feeney illustrated with their simulation.

Figure 9 shows the difference between the model of mortality change described in this section, and a linear shift model in which mortality follows a Gompertz schedule.

An interesting feature of these two models is that they produce either: (1) exact linear change in both CAL and LCLE in the case of the linear shift assumption; (2) approximate linear change in both CAL and LCLE in the case of the Gompertz model with constant log-linear decline.

Getting back to Figure 3 showing the correspondence between CAL_{60} and $LCLE_{60}$ among Swedish females, we can see that the correspondence holds in a context where the two model patterns of mortality change do not apply. In this population, age specific death rates have not been declining at a constant, log-linear rate during the period of interest. In fact, there was a stark acceleration in the mortality decline around 1945, producing a sharp increase in period life expectancy at age 60. Also, CAL does not change linearly during the period, implying that the linear shift assumption does not hold either. Nonetheless, the correspondence remains excellent. This means that there must be other patterns of mortality change, besides the model patterns described above, that can produce a precise correspondence between LCLE and CAL.

IV. Simulations

In order to study how the correspondence holds more generally, aside from the two model patterns discussed in the above section, we turn to simulations. For these simulations, we use the classic Lee and Carter model of mortality change:

$$\ln(m(x,t)) = a(x) + b(x)k(t) \tag{13}$$

This model of mortality change is convenient for our purpose. It is a flexible model which will allow us to simulate various patterns of mortality change and examine the extent to which the CAL-LCLE correspondence is resistant to these patterns.

For these simulations, we use mortality above age 60 among Swedish females as a starting point. We fit a Lee and Carter model to these data, and obtain estimates for the function a(x), b(x) and k(t). We find that a(x) and b(x) are approximately linear in the observed data. We thus use a linear version of these functions in our simulations. Observed and linear versions of a(x) and b(x) are shown in Figure 10.

Our simulations focus on k(t) and its impact on the CAL-LCLE correspondence. Indeed, in actual populations, a(x) and b(x) tend to vary relatively little in shape, while k(t), representing time trends in period mortality rates, can follow many different trajectories. In this paper, we simulated 24 trajectories of k(t), representing drastically different scenarios of mortality change. These simulated scenarios of k(t) are shown in Figure 11.

Figures 12a-12f show CAL_{60} and $LCLE_{60}$, along with period e_{60}^{p}, in these 24 scenarios of mortality change. This figure shows that the correspondence holds extremely well in most cases. This is especially true when mortality decreases or increases monotonically. The only scenarios for which there is some discrepancy is when mortality changes abruptly from decreasing to increasing, or vice-versa. But even in these extreme cases, the discrepancy appears only around the time of the mortality reversal, and the correspondence is reestablished soon after.

These simulations show that in a Lee-Carter model of mortality change above age 60 in which a(x) and b(x) are modeled after Swedish females, the CAL-LCLE correspondence is very robust to various patterns of mortality change (embodied in k(t)). These simulations go beyond the linear shift model or Gompertz model discussed in the previous section, and extend the range of situations in which we find a precise CAL-LCLE correspondence.

V. Discussion

We find that the correspondence between CAL and LCLE holds exactly or approximately in a number of situations. It holds exactly under the constant shift assumption. It holds approximately under the Gompertz model with log-linear decline. The correspondence also holds rather well in actual populations when considering mortality above 30, and extremely well for mortality above age 60. This correspondence takes place even when patterns of mortality change depart substantially from the constant-shift or Gompertz-log-linear-change assumptions. Finally, we find that the correspondence holds well in Lee and Carter simulations with various k(t) trajectories.

The accuracy of the correspondence, especially at age 60, but also to some extent at age 30, leads to the following conclusions:

1) When considering mortality above age 60 (or 30), CAL can be interpreted as the mean age at death for the cohort currently reaching its life expectancy. This correspondence provides an additional interpretation to this indicator which already has a number of known properties.

2) CAL may be a useful device for projecting cohort life expectancy, especially above age 60. Specifically, CAL_{60} may be used for estimating the life expectancy at age 60 for the cohort reaching age 60 at time t- CAL_{60}(t). CAL_{60}(t) can be calculated without forecasting mortality, while the calculation of e_{60}^{c}(t- CAL_{60}(t)) requires forecasting. In Figure 3, given the historical correspondence between CAL and LCLE, it seems reasonable to assume that $LCLE_{60}$ will follow the trajectory indicated by CAL_{60}. More research is needed to assess whether forecasting cohort life expectancy using CAL simply amounts to a log-linear projection of mortality rates, or whether is provides a shortcut for more complex methods of mortality forecasts.

As we said in the introduction, the correspondence between CAL and LCLE was first observed in the literature on tempo effects in mortality. However, the existence of a correspondence between CAL and LCLE does not per se demonstrate the existence of tempo effects, nor does it justify the use of CAL or related measures to correct for tempo bias. The correspondence only shows that in a number of populations, mortality has evolved in such a way such that LCLE, an interesting index in its own right, is relatively close to CAL. In order to use CAL as a proxy for LCLE in a correction for tempo bias, one would first have to demonstrate that the current value of LCLE provides an indicator of period mortality conditions free of tempo effects, or equivalently, that the discrepancy between period life expectancy and LCLE does indeed amount to tempo effects.

Guillot (2006) argued that, under a specific mortality scenario in which new mortality conditions appearing during a given year produce fixed delays in future cohort deaths, CAL immediately adjusts to the ultimate level of cohort life expectancy and may thus better reflect the underlying mortality conditions than period life expectancy during that year. This single-shift scenario (or life-extension pill scenario) is represented in Figure 13. Under such a scenario, while CAL adjusts immediately, cohort life expectancy (and thus LCLE) adjusts only gradually to this ultimate level. (This is also shown by Rodriguez (2006) and Goldstein (2006).) In particular, the level of LCLE anticipates the change in mortality conditions and start increasing before these new mortality conditions appear. In this scenario, the CAL-LCLE correspondence does not hold, and it is CAL, rather than LCLE, that correctly indicates the underlying mortality conditions. This means that even in this particular scenario where the change in underlying conditions is known, LCLE is not an adequate indicator of these underlying conditions. We did not expect the CAL-LCLE correspondence to hold in this particular scenario. However, we do note that, in this theoretical example which has been used to demonstrate the existence of tempo bias, LCLE does not reflect underlying mortality conditions.

Nonetheless, LCLE is useful in many other respects. Cohort life expectancy is an indicator that summarizes the mortality experience of an actual cohort. If life expectancy increases from one cohort to the next, this undoubtedly means that the underlying cohort mortality conditions have improved. The same is not true of period life expectancy. For example, given the influences of childhood conditions on later-life mortality, an increase in period life expectancy from one year to the next may be due in part to improved conditions for children many years earlier, rather than to actual improvements in period mortality conditions. The clear correspondence between cohort life expectancy and underlying cohort mortality conditions is the strength of this indicator. However, its weakness is the fact that it represents many years of mortality influences. Associating cohort life expectancy with its corresponding mean year at death is a useful way of summarizing these many years of mortality influences, spanning the entire life course of that cohort. However, average year at death for a given cohort appears to have little to do with the underlying mortality conditions of that year.

While LCLE provides a useful way of representing cohort mortality, and while CAL can be used as a shortcut for this representation above age 60, we find that the correspondence between CAL and LCLE does not hold well for life expectancy at birth. This implies that when it comes to estimating lagged cohort life expectancy at birth, CAL should not be used as a proxy. This means that there will be no alternative to mortality forecasts for estimating the current value of LCLE. This diminishes the attractiveness of this indicator.

Although the interpretation of LCLE and CAL as tempo-adjusted mortality measures is debatable, the main finding of this paper – that the LCLE-CAL correspondence holds at older ages in a number of situations – remains. This correspondence has implications beyond the issue of tempo bias, including in the area of mortality forecasts.

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Figure 1: Correspondence between CAL_0 and $LCLE_0$ among Swedish females

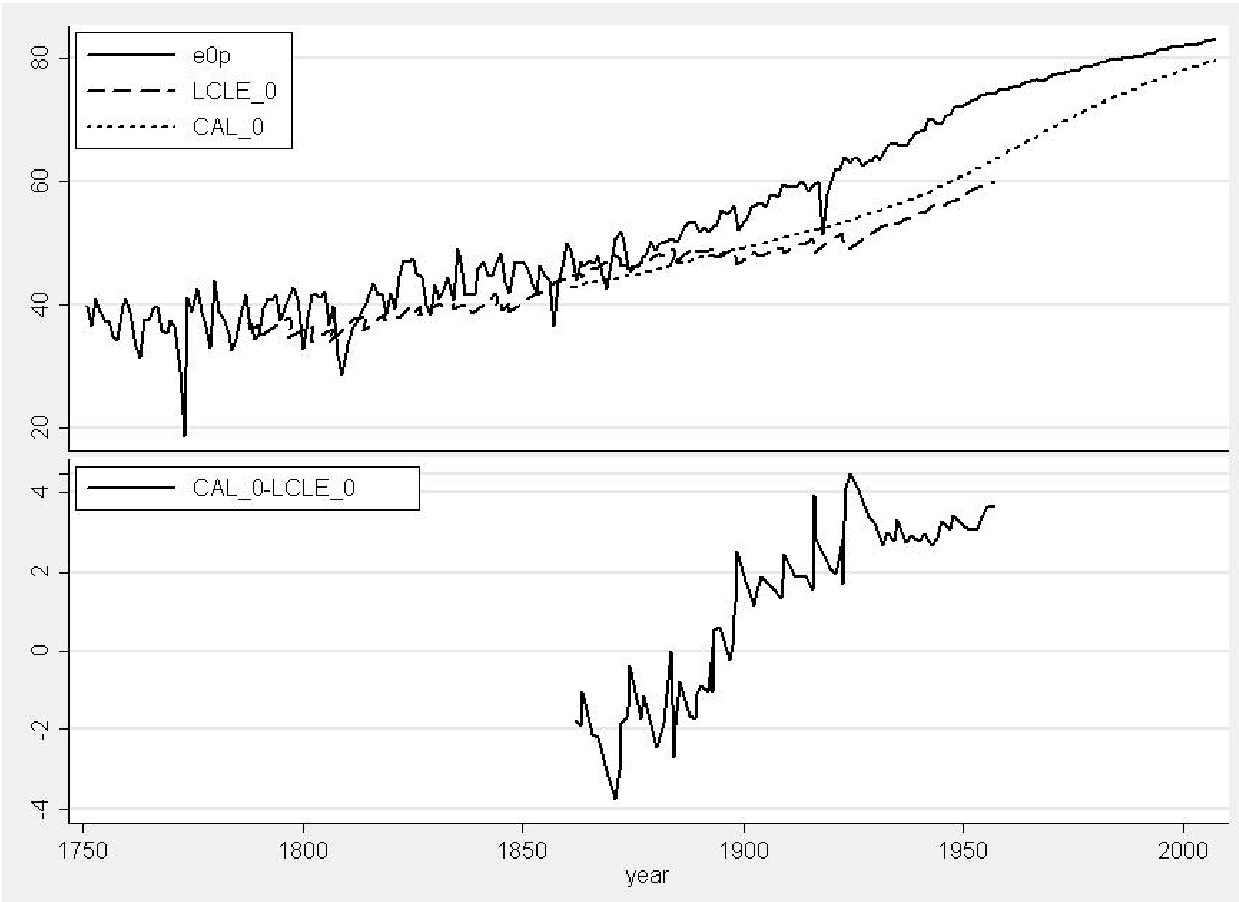


Figure 2: Correspondence between CAL_{30} and $LCLE_{30}$ among Swedish females

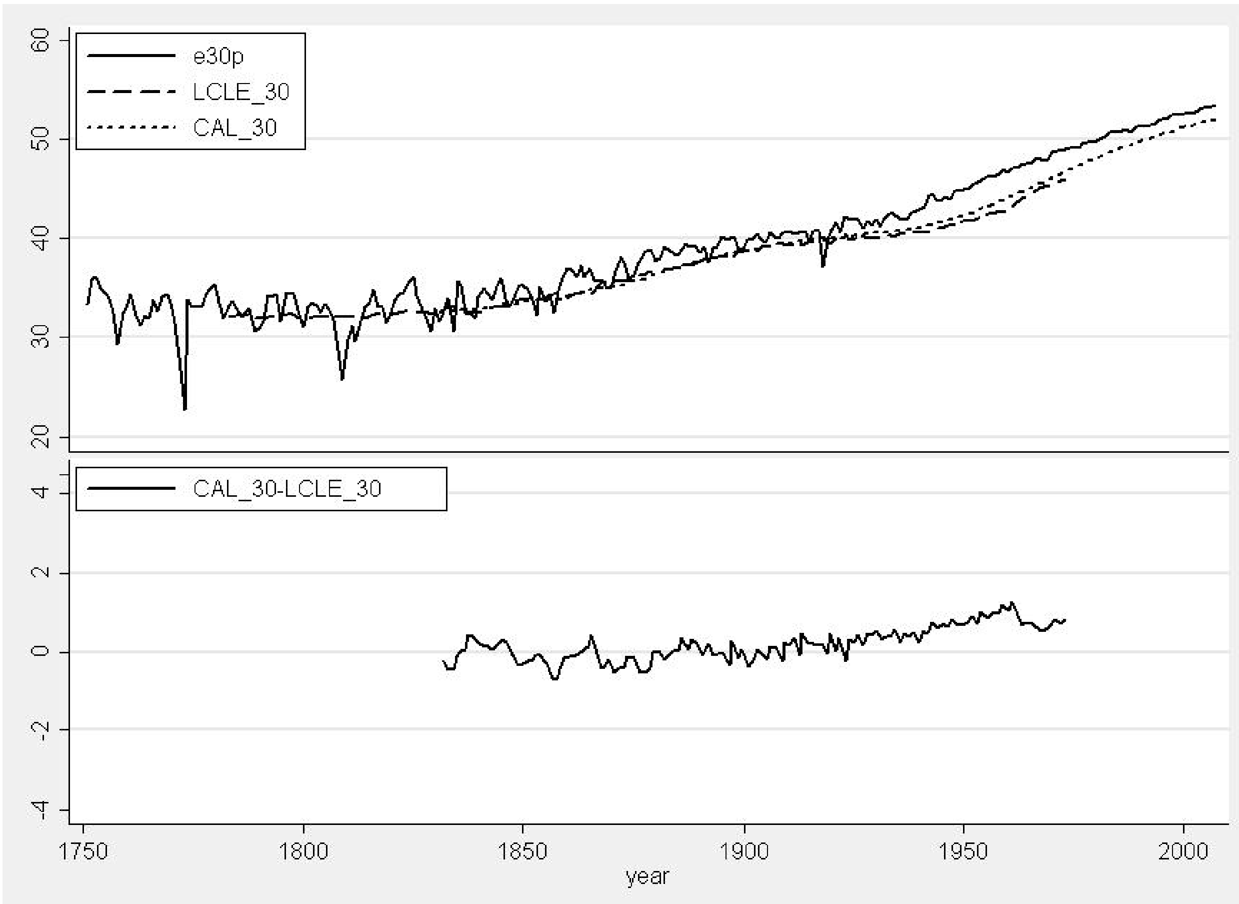


Figure 3: Correspondence between CAL_{60} and $LCLE_{60}$ among Swedish females

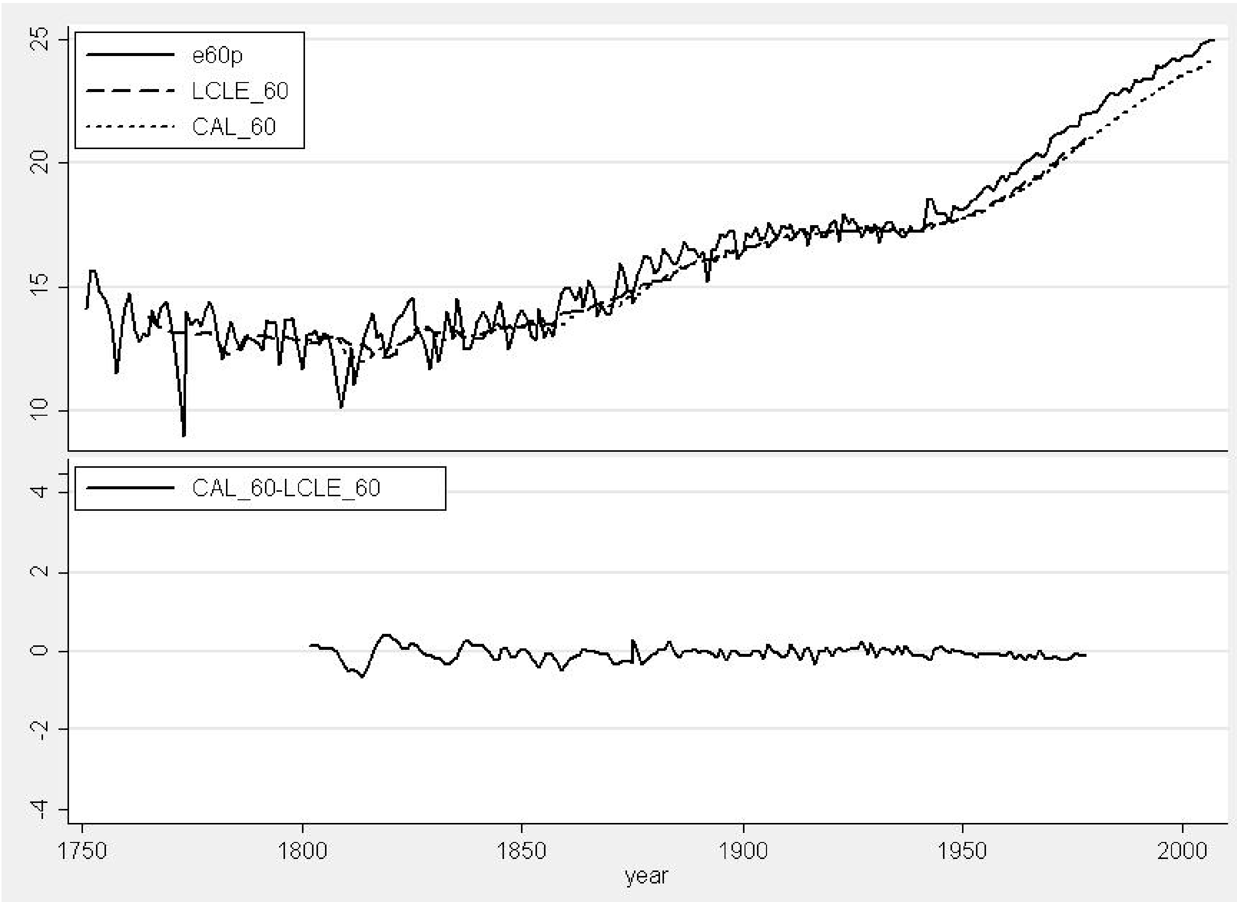


Figure 4: CAL/LCLE ratio among Swedish Females.

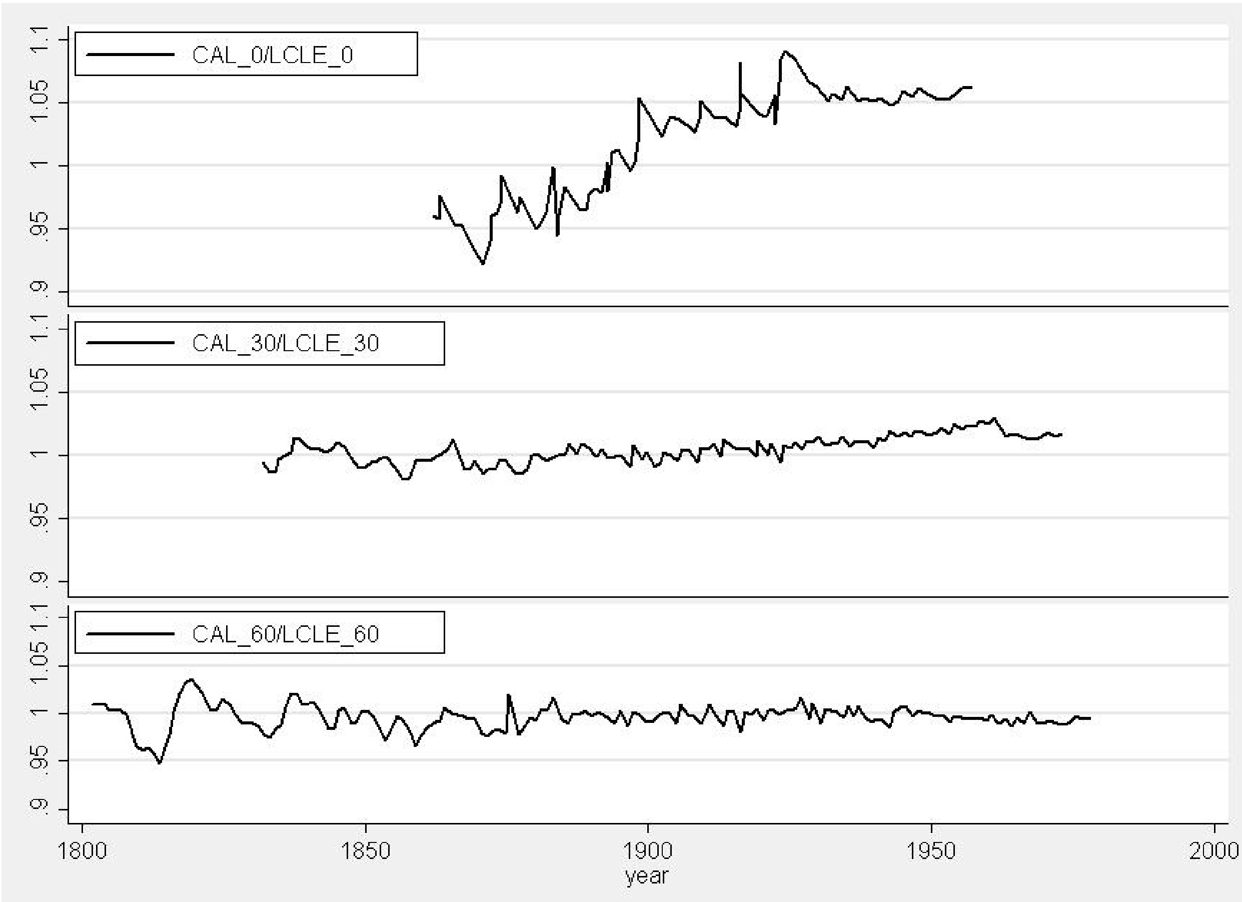


Figure 5: $CAL_0/LCLE_0$ ratio among countries in the human mortality database.

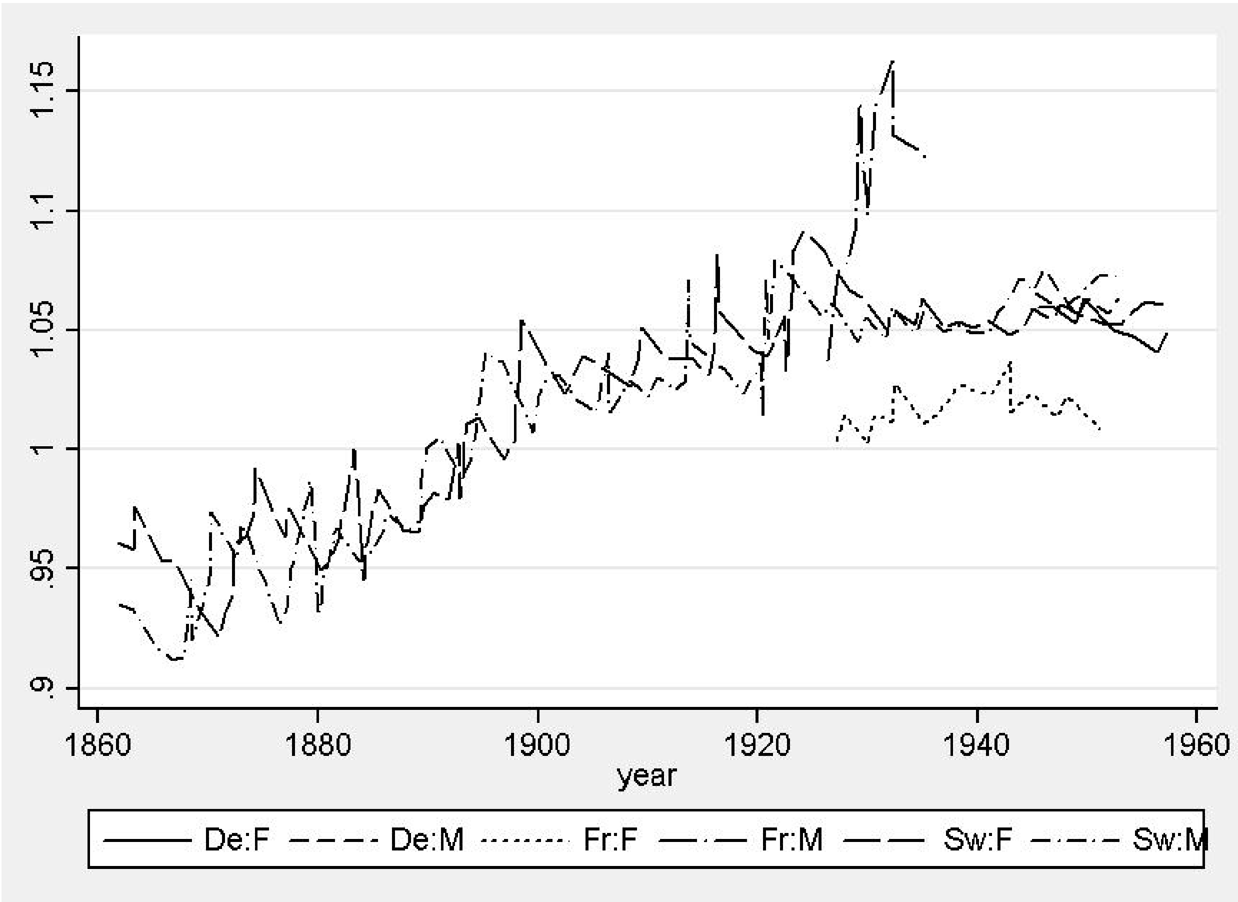


Figure 6: $CAL_{30}/LCLE_{30}$ ratio among countries in the human mortality database

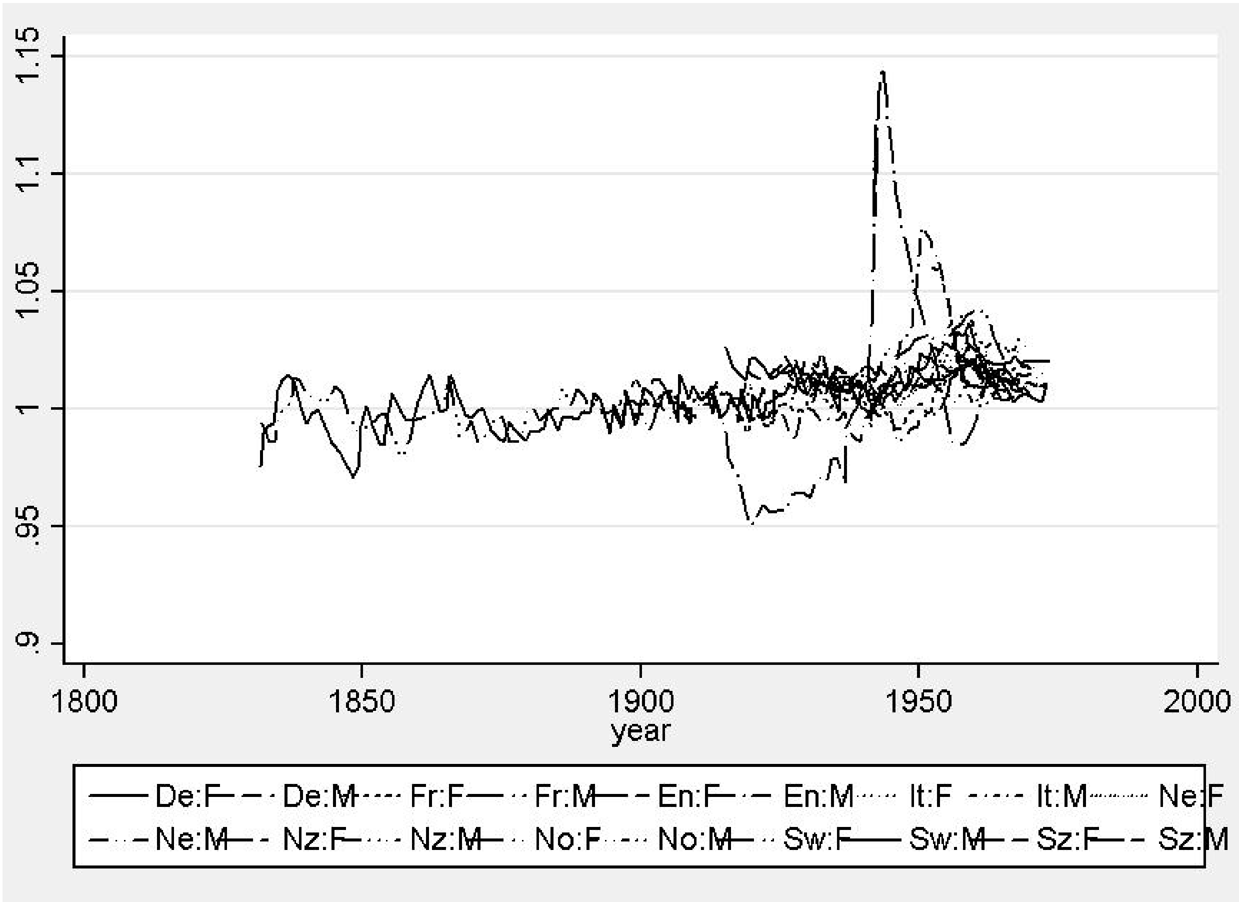


Figure 7: $CAL_{60}/LCLE_{60}$ ratio among countries in the human mortality database.

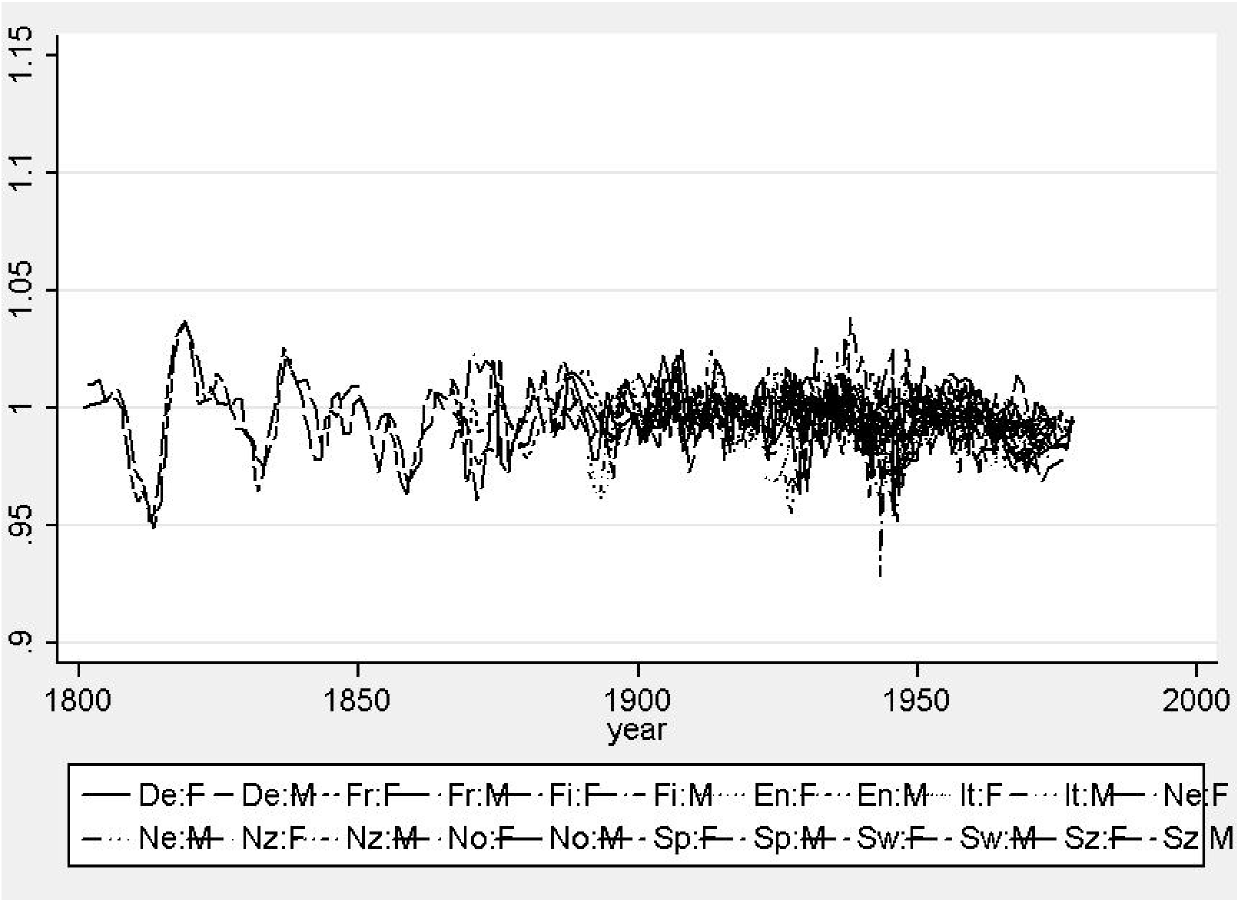


Figure 8: Linear shift pattern of mortality change

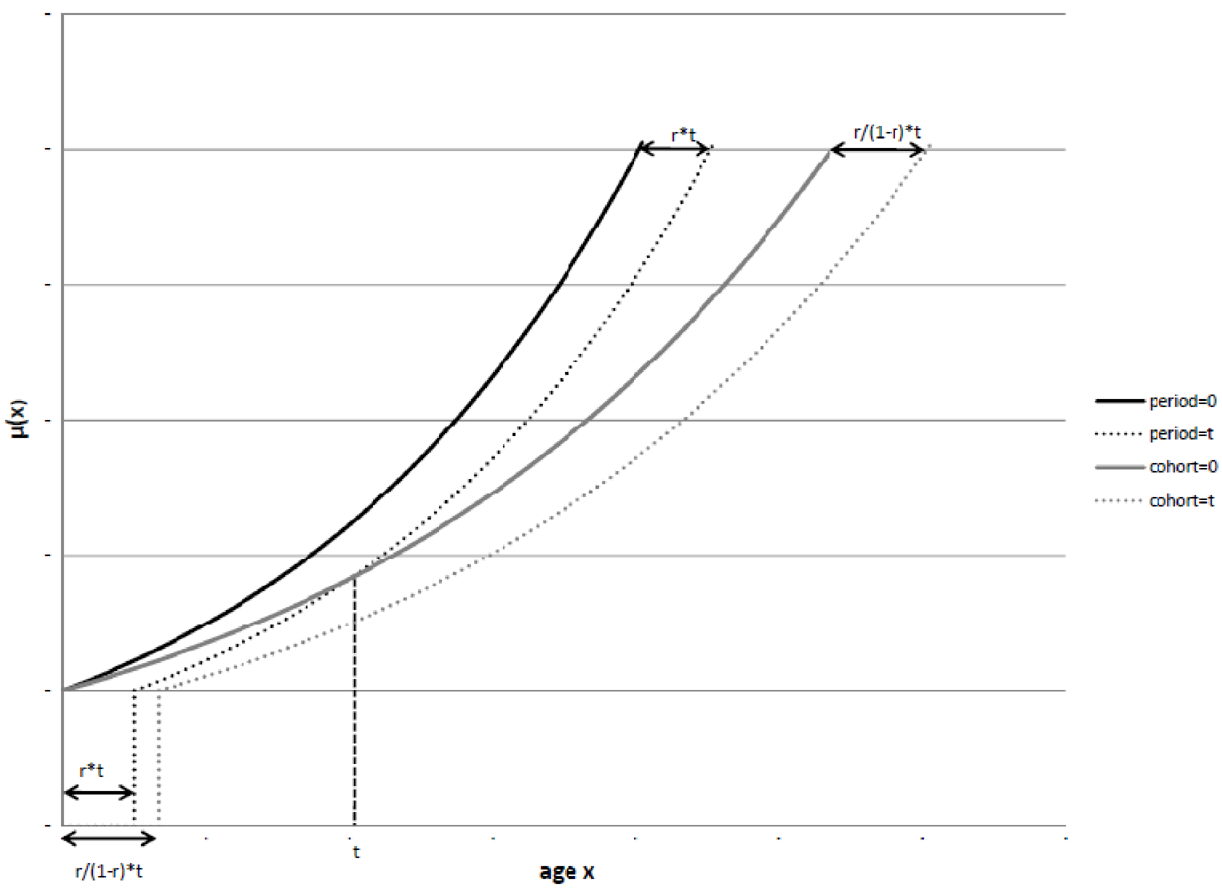


Figure 9: Gompertz pattern of mortality with log-linear decline, and its correspondence with the linear shift pattern

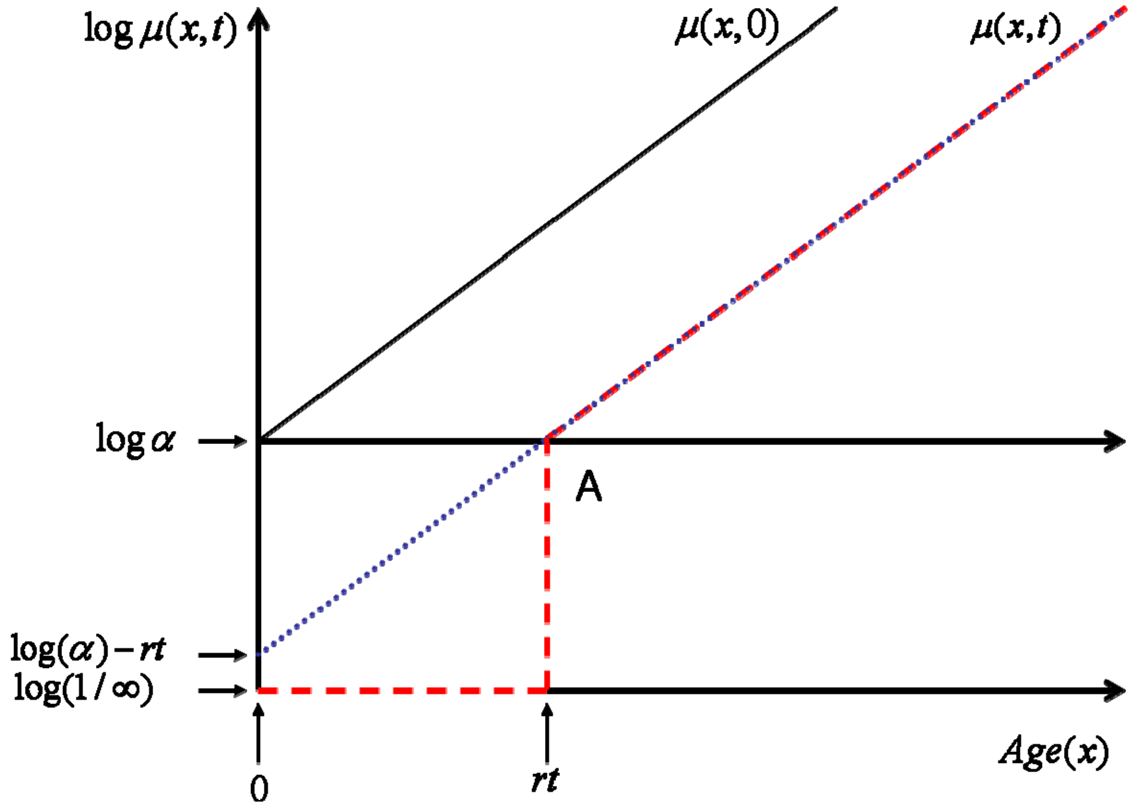
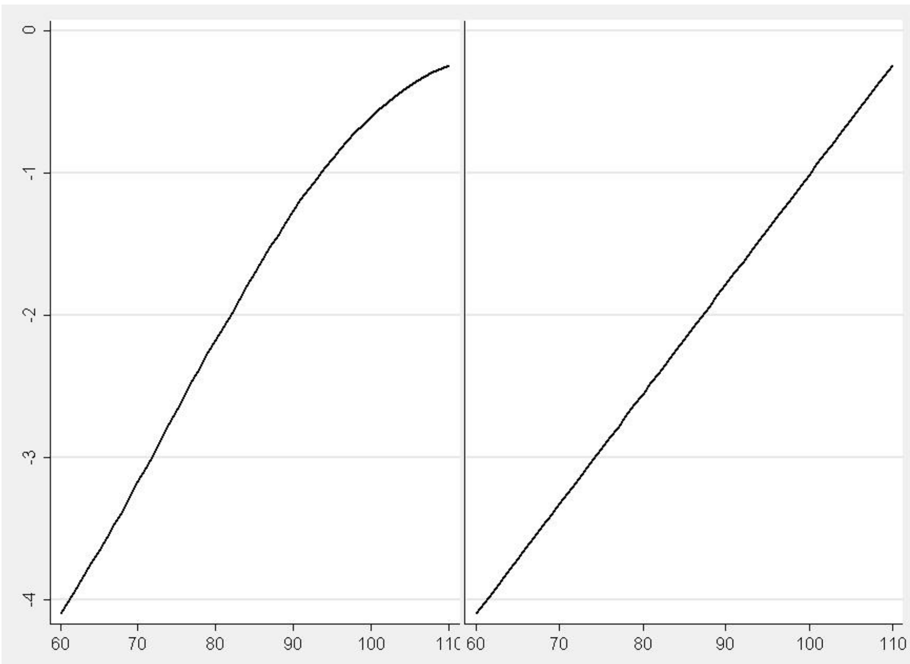


Figure 10: Observed vs. linear pattern of Lee-Carter's a(x) and b(x) among Swedish females (ages 60 and above).

a(x):



b(x):

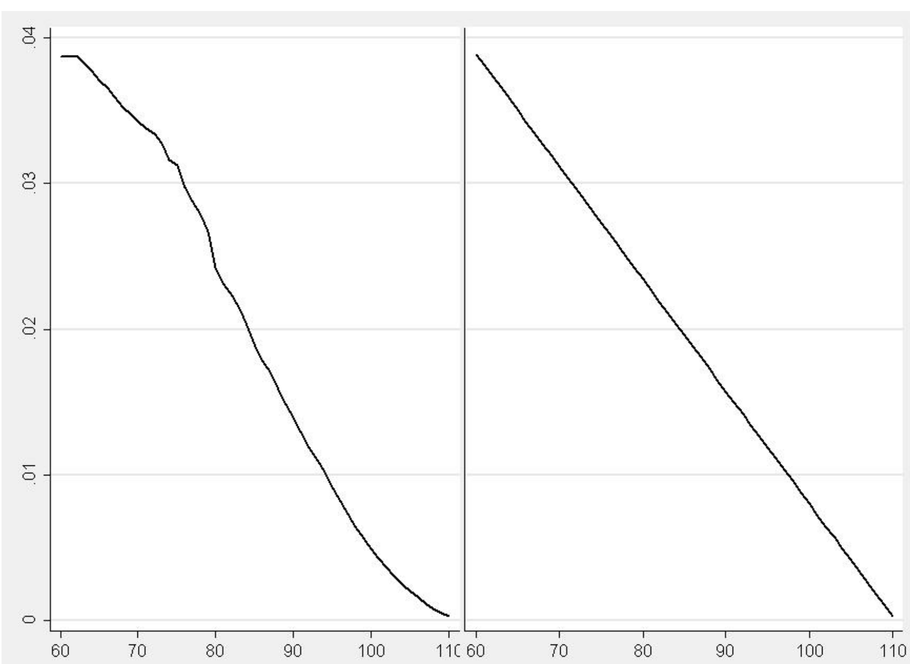


Figure 11: Scenarios of k(t) trajectories in simulations of mortality change

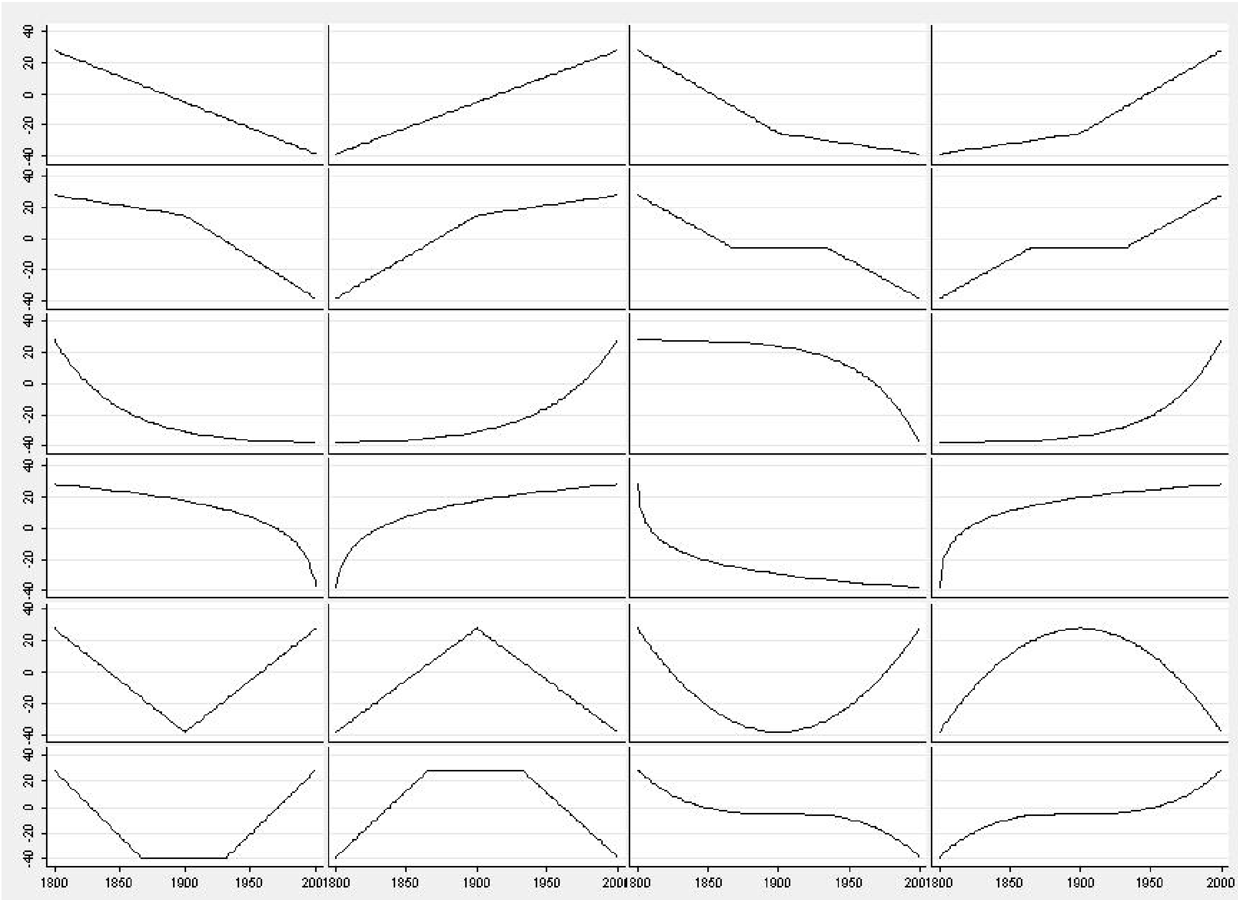
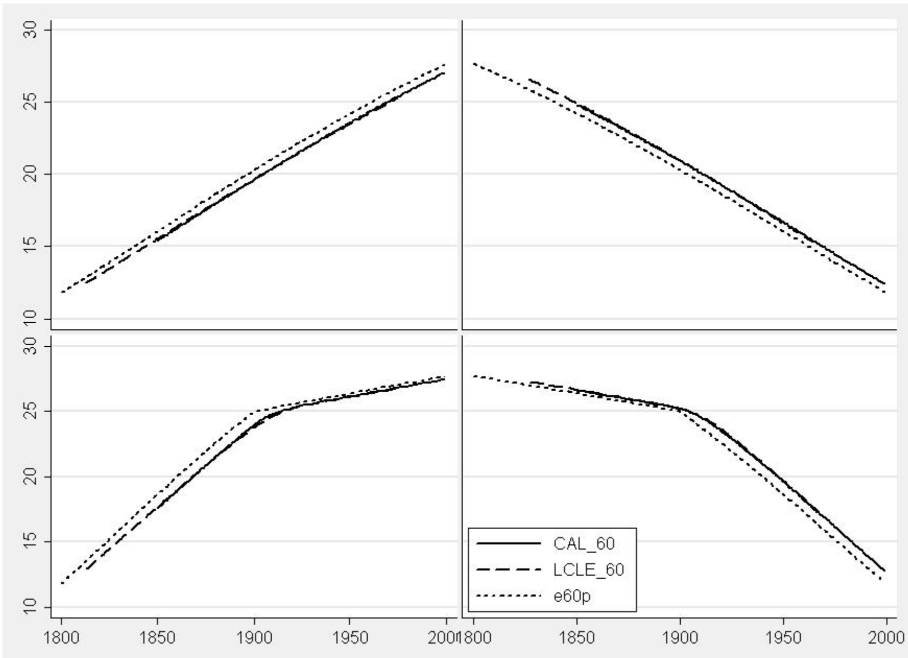


Figure 12: e_{60}^{p}, CAL_{60} and $LCLE_{60}$ in simulated scenarios of mortality change.

a) scenarios 1-4



b) scenarios 5-8

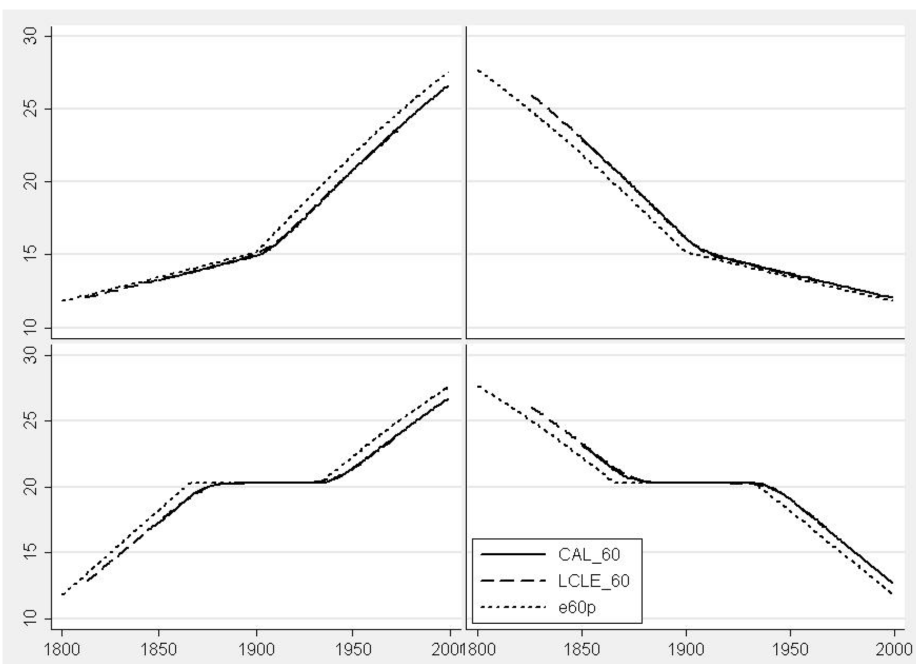
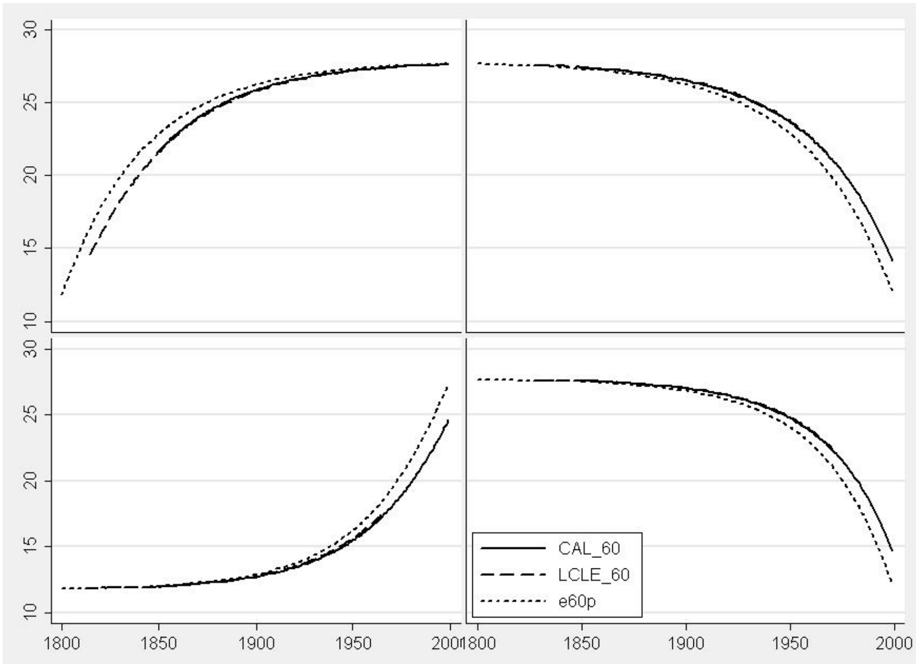


Figure 12 (cont'd):
c) scenarios 9-12



d) scenarios 13-16

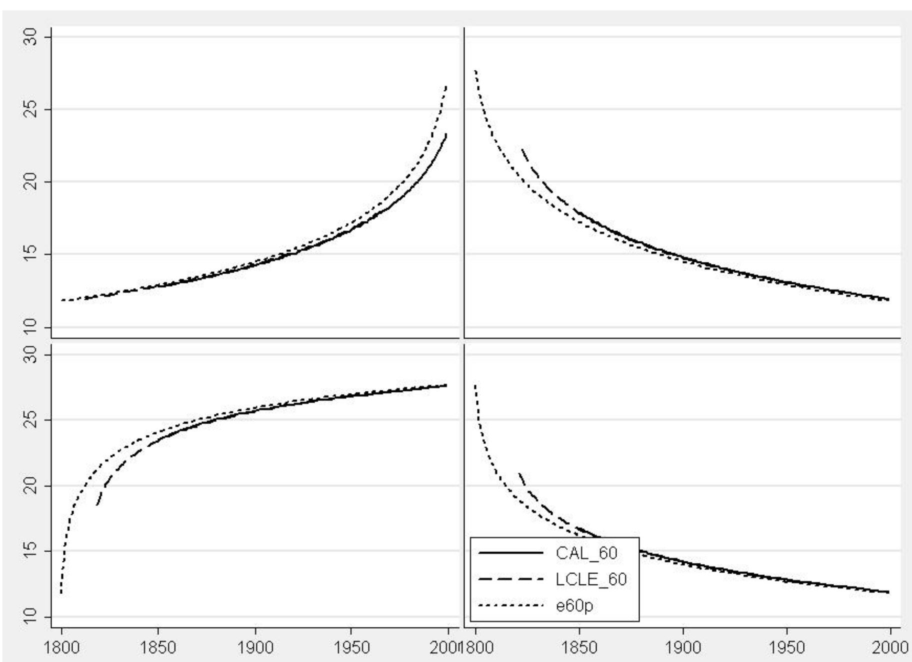
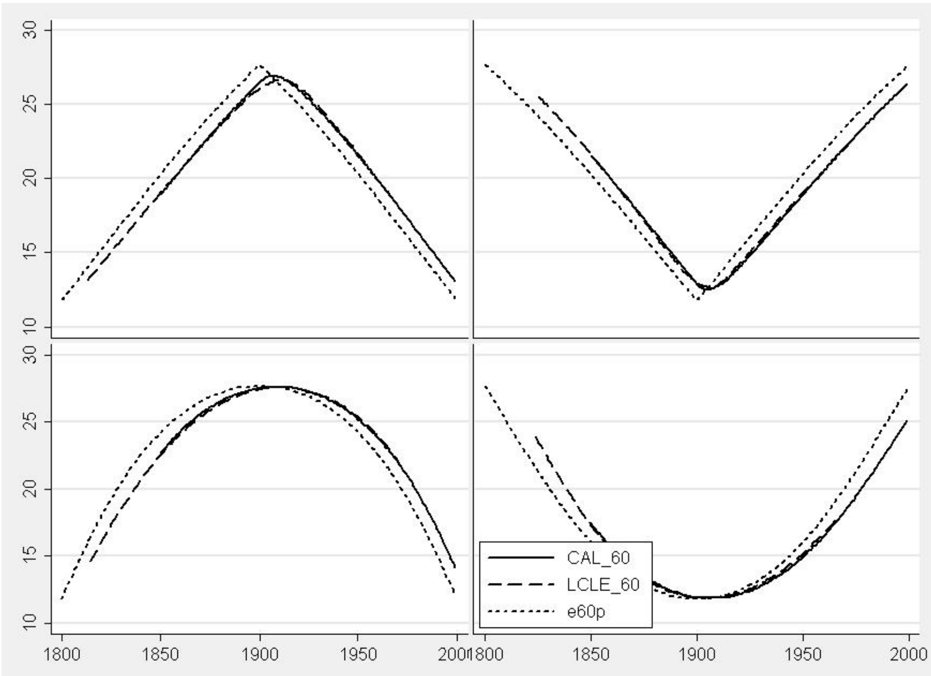


Figure 12 (cont'd):
e) scenarios 17-20



f) scenarios 21-24

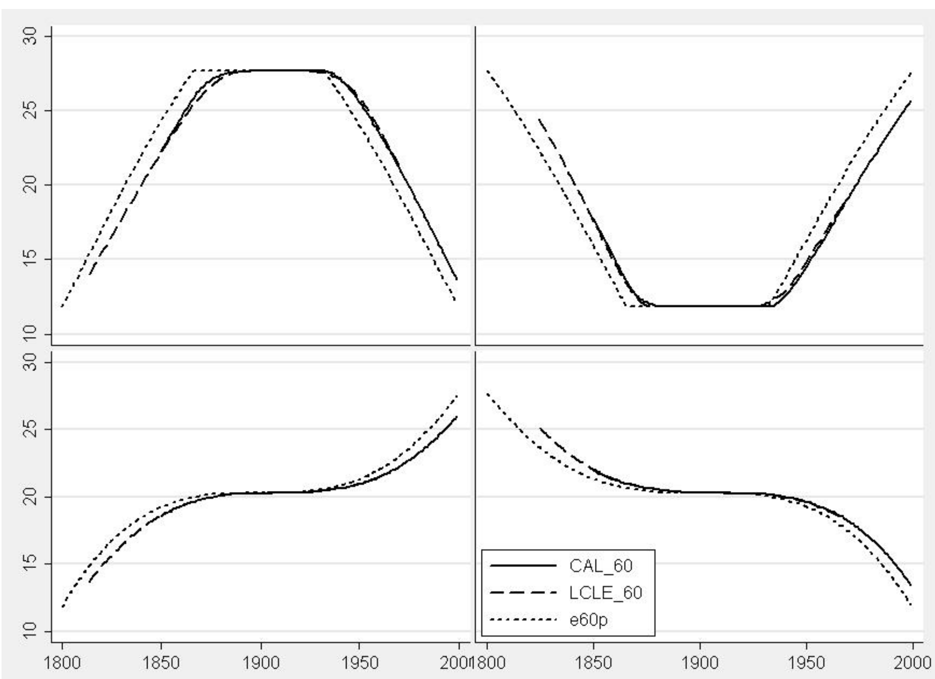


Figure 13: e_0^p, CAL_0 and $LCLE_0$ in Bongaarts and Feeney's single shift scenario.