Shifting mortality: inevitable or anomaly?

Sarah Zureick*

March 2, 2010

Abstract

While shifting mortality was conceptualized as a shift in mortality rates across age, trends in variability of age at death are used to evaluate whether or not a country is exhibiting signs of shifting mortality. In this paper, we reexamine the relationship between mortality change and trends in variability age of death in order to evaluate whether shifting mortality is an inevitable outcome of the mortality transition. Through simulation exercises, perturbation analysis, and decomposition analysis, we arrive at a better understanding of how initial mortality conditions and the age-pattern of mortality change interact to produce mortality compression, expansion, or shifting. We find that proportional mortality change that is fixed across ages does not necessarily lead to a parallel shift in the death distribution. In addition, we demonstrate that certain initial mortality conditions are particularly primed for compression.

*Department of Demography, University of California, Berkeley
Please send comments to sarahz@demog.berkeley.edu

Reviewing the historical mortality experience of human beings is a grim endeavor. For most of history, humans experienced relatively short average life times, with many not living past infancy, and high variability of age at death. Fortunately, focusing on human mortality in the modern era is not quite so bleak. While we are still bound by the chains of mortality, due to the considerable progress made against mortality in recent times, we now enjoy greater longevity and more certainty about the timing of death.

This paper focuses on the latter mortality advantage that has arisen as a result of the mortality transition: greater certainty about the timing of death. Not only are we living longer, but the ages at which the majority of us die have become increasingly compressed. Much attention has focused on documenting trends in mortality compression in response to Fries claim that mortality would become increasingly compressed as humans approached a fixed biological limit to life span [Fries, 1980]. Observations of historical trends in mortality revealed that the once rapid pace of mortality compression has waned and in some cases stagnated or reversed while life expectancy continues to rise [Myers and Manton, 1984a, Myers and Manton, 1984b, Wilmoth and Horiuchi, 1999, Cheung and Robine, 2007]. These observations provide evidence against the approach to a fixed limit in life span and characterize a possible new era of mortality change: the shifting mortality era.

In this paper, we explore in depth the concept of shifting mortality. First, we provide a review the current literature on this topic and find that there is a disconnect between the conceptual roots and the measurement of shifting mortality. While shifting mortality was conceptualized as a movement of age-specific mortality rates across age over time, the trend towards shifting mortality is assessed by measuring variability of age at death in the death distribution. In this paper, we aim to better understand how changes in age-specific mortality rates, especially proportional changes fixed across age and time, affect the dispersion of the death distribution. This relationship is not as simple as one might hope. By focusing on the interrelationship between the age pattern of mortality change and initial mortality conditions, this paper provides new insights into mortality compression and the development of shifting mortality.

1 Current understanding of shifting mortality

A number of researchers have documented the emergence of shifting mortality; however, the timing of the emergence of shifting mortality and the specific geographic areas where shifting mortality has been observed vary depending on the measure of mortality compression used in the analysis. These same problems of different interpretations depending on the choice of measure emerged in the discussions surrounding mortality compression and possible fixed limits to life span [Fries, 1984, Myers and Manton, 1984b, Robine, 2001]. Both Wilmoth and Horiuchi as well as Cheung et al. have provided overviews of the measures available to assess mortality compression and rectangularization of the survival curve [Wilmoth and Horiuchi, 1999, Cheung et al., 2005]. In this paper, we offer an overview of measures variability of age at death that have been used to detect shifting mortality. All of these measures capture some type of variability in the death distribution. Differences between these indicators

arise because of differences in the age range covered by the measures, differences in the central indicator used in the calculation of dispersion (mean versus mode), and whether or not the measurement is attached to fixed percentiles of the death distribution:

- IQR-inter-quartile range, age span between the 25th and 75th percentiles of the cumulative death distribution [Wilmoth and Horiuchi, 1999].
- $C50$-age span corresponding to the most compressed 50 percent of deaths from the death distribution [Kannisto, 2000].
- S_{10}-standard deviation of ages at death above age ten [Edwards and Tuljapurkar, 2005].
- SDM-standard deviation around the modal age at death [Canudas-Romo, 2008]
- $SD(M+)$-standard deviation above the modal age at death [Kannisto, 2000, Kannisto, 2001, Cheung and Robine, 2007]

In addition to examining traditional measures of variability of age at death, we also include in our analysis the life span disparity measure $e^\dagger$. This measure, recently proposed by Zhang and Vaupel, indicates the person years lost in the life table due to early death: $e^\dagger = \int_0^w e_x d_x$, where e_x is life expectancy at age x, d_x indicates the number of deaths that occur at age x, and w is the oldest age group.

- $e^\dagger$-a measure of life span disparity [Zhang and Vaupel, 2009].

Trends in these different measures over course the 20th century in France are shown in Figure 1. As can be seen from the figure, these different measures offer different interpretations of the history of mortality compression in France. Relative to the massive compression of mortality indicated by trends in IQR during the period 1900-1950, the trends in IQR in the second half of the century seem to indicate a transition to shifting mortality as Wilmoth and Horiuchi also observed in Sweden [Wilmoth and Horiuchi, 1999]. Kannisto and Robine have argued that IQR over exaggerates the pace of mortality compression because it sometimes measures the space between the two modes in the death distribution rather than the concentration of deaths in one or the other. These authors have shown that documenting trends in variability of age at death using the measure $C50$ rather than IQR reveals that compression was still occurring in a number of countries throughout the latter half of the twentieth century [Kannisto, 2000, Robine, 2001].

Consistent with the assessment of Edwards and Tuljapurkar of trends in S_{10} in industrialized countries, trends in this measure for France indicate stagnation since the 1950's [Edwards and Tuljapurkar, 2005]. In contrast to the trends in S_{10}, trends in SDM, S_0, and $e^\dagger$ indicate continued compression during the second half of the twentieth century reflecting gains made against infant mortality. Unlike the other measures, $SD(M+)$ does not begin steadily declining in France until the latter half of the twentieth century.

While these different measures produce different interpretations of the geographic coverage and timing of the transition to shifting mortality, the authors utilizing these different measures to study trends in mortality compression cite common sources as the originators of the concept of shifting mortality. This concept was first introduced by Kannisto in his study of trends in old age mortality. One measures the "the age shift in mortality" introduced by Kannisto by finding the difference between the ages

that correspond to the same mortality level between two time points. For instance, in the example given by Kannisto, q_{85} for Austrian females in the period 1950-1960 is 0.1721. To find the age shift in mortality from 1950-1960 to 1960-1970, one tries to find x in the period 1960-1970 where $q_x = 0.1721$. This occurs at $x = 85.96$. Thus, the age shift in mortality is 0.96 years [Kannisto, 1996].

As will be illustrated through "mortality experiments" in the later sections of this paper, even if all age-specific mortality rates shifted across age .96 years between these two time periods, the result would not necessarily imply shifting mortality when using the measures of mortality compression described above. The translation of a shift in mortality rates to a shift in the death distribution requires special initial mortality conditions. For instance, if the age-specific mortality curve can be described as a Gompertz curve, a proportional decrease in mortality rates fixed across age and time corresponds exactly to a shift in mortality rates across age. Under these circumstances, the death distribution retains its shape while shifting to higher ages, and thus the measures of mortality compression described above all exhibit constant trends. This ideal shifting mortality scenario is illustrated in Figure 2.

Bongaarts proposal of the shifting logistic model provided further foundation for the concept of shifting mortality [Bongaarts, 2005]. In fitting the following logistic model to annual mortality rates for ages 25+ from a variety of countries included in the Human Mortality Database, Bongaarts notes that the slope parameter, β, has been relatively stable in the period 1950-2000:

$$\mu(x,t) = \frac{\alpha(t)e^{\beta(t)x}}{1+\alpha(t)e^{\beta(t)x}} + \gamma(t) \tag{1}$$

The relative stability in the β parameter allows a new interpretation of recent changes in senescent mortality rates. As Bongaarts states, "instead of interpreting mortality as rising or falling, the schedule of the force of senescent mortality can be viewed as shifting to higher or lower ages over time" [Bongaarts, 2005, p. 29]. In adapting the model for forecasting, Bongaarts allows the α and γ parameters vary with time while the β parameter is fixed [Bongaarts, 2005].

Bongaarts bases his model of shifting mortality on data from the second half of the twentieth century. Therefore, one might ask whether the period of observation covered by Bongaarts data should be considered the shifting mortality era. As can be seen in Figure 1, none of the measures of mortality compression, except for S_{10}, indicate a transition to shifting mortality in France as early as 1950. Of course, Bongaarts only describes senescent mortality as shifting, not all-cause mortality. A continued decline in the background mortality, as described by the background parameter, $\gamma(t)$, might explain why measures of mortality compression that cover most of the adult age range such as $C50$, $e^{\dagger}$, IQR, and SDM still exhibit compression during the 1950s. What appears contradictory, however, is the continued declines in $SD(M+)$ since the 1950s observed in many of the same countries included in Bongaarts' analysis [Kannisto, 2000, Kannisto, 2001, Cheung and Robine, 2007, Robine, 2008, Thatcher et al., 2008] .

SD(M+) should largely reflect trends in senescent mortality, which overwhelms background mortality above the modal age. Consider the senescent component of the Bongaarts shifting logistic model, which is the first term of equation 1. As Thatcher et al. have demonstrated, the death rates and modal age at death implied by this model

depend on both the α and β parameter; however, compression around the modal age is determined solely by the β parameter. Therefore, trends in the measure SD(M+), can be determined just by looking at trends in the β parameter [Thatcher et al., 2008]. As noted above, Bongaarts finds trends in the β parameter to be remarkably consistent across his period of observation. This discrepancy between consistency in the β parameter, which implies shifting mortality, and observed declines in SD(M+), which likely reflects compression in senescent mortality at older ages is troubling.

While Kannisto and Bongaarts introduced the concept of shifting mortality as a shift in age-specific mortality rates across age, those documenting the transition to the shifting mortality era describe shifting mortality as occurring when the death distribution shifts to higher ages while retaining the same shape (i.e. in the case of $SD(M+)$, the modal age at death rises while retaining the same variability of age at death above the mode as measured by $SD(M+)$ [Cheung and Robine, 2007]). For the shifting logistic model proposed by Bongaarts, the shift in senescent mortality across age should result in a shift in the senescent death distribution. It is not true generally, though, that a shift in mortality across age leads to a shift in the death distribution. This is described analytically in the next section.

Researchers have made efforts to understand exactly how underlying changes in age-specific mortality produce shifting mortality. In an attempt to understand whether an increase in the modal age at death always implies mortality compression, Thatcher et al. fit the following logistic model to death rates at age 70 and 90:

$$\mu(x) = \frac{\alpha e^{\beta x}}{1 + \alpha e^{\beta x}}$$

The authors find that the modal age at death depends both the α and β parameters while the measure of mortality compression around the mode depends solely on the β parameter (as mentioned above). Mortality compression occurs if the logit of the death rate at age 70 falls faster than the logit of the death rate at age 90. To achieve shifting mortality, where the modal age increases while variability of age at death above the mode remains constant, the logits of the death rates at age 70 and 90 must fall at the same rate so that the β parameter retains the same value over time. Fitting their historical model for six countries included in the Human Mortality Database, Thatcher et al. observe a recent trend towards a stable β, and thus shifting mortality, in French females, Italian males, and both males and females in Japan. An important lesson to take from this analysis is that if old-age mortality can be described accurately by a logistic model fit to mortality rates at 70 and 90, similar rates of decline in mortality at these two ages is a necessary condition for observing shifting mortality under this model. This suggests that the eventual transition to a shifting mortality era is dependent on similarity or dissimilarity in the age pattern of mortality decline.

Canudas-Romo explores the emergence of the shifting mortality era by simulating mortality change over time using a variety of mortality models and comparing the results to actual trends in the modal age at death, number of deaths at the modal age, and SDM for a number of countries included in the Human Mortality Database [Canudas-Romo, 2008]. By extrapolating age-specific mortality rates using both the Gompertz and logistic models of mortality change as well as the more complex Siler model, Canudas-Romo shows that the number of deaths at the modal age and the stan-

dard deviation of the distribution of deaths around the modal age eventually converge toward an asymptote while the modal age continues to rise. These results are based on assuming a fixed relationship of mortality over adult ages as implied by the fixed β parameters in the Gompertz mortality, logistic model, and Siler mortality change models.

The results of Canudas-Romo's simulation using the Siler mortality change model indicate that the pace of mortality compression slows after infant mortality has reached a relatively low level. This suggests that the potential for compression depends on current mortality conditions. We explore this further in this paper along with the effect of similarity or divergence in the age-pattern of mortality change.

An explanation for shifting mortality related to divergence in the age pattern of mortality change appears in Wimoth and Horiuchi's 1999 piece on trends in variability of age at death [Wilmoth and Horiuchi, 1999]. Using historical mortality evidence from Sweden, the authors document that life expectancy rose steadily throughout the period of observation (1751-1995). In contrast, the IQR remained relatively stable from 1751-1875, declined rapidly during the period 1876-1955, and then remained relatively stable at its new lower level from 1955 onwards. The authors observe that during the period of rapid decline in IQR, the average annual rate of proportional mortality decline varied greatly across age with much more rapid progress at younger ages. In contrast, during periods in which the IQR remained relatively stable, the average annual rate of mortality decline was similar across ages[1]. This observation along with some intuitive reasoning led the authors to propose that dissimilarity across the age pattern of mortality change was the main cause of the decrease in IQR.

The notion that divergent age patterns of mortality change (with younger age groups experiencing faster progress) lead to a decline in the variability of age at death is especially appealing if it also true that if the age pattern of change in mortality is similar across ages (especially the ages where deaths are concentrated), then variability of age at death will remain constant. The results of Thatcher et al. seem to suggest that similarity in the age-pattern of mortality change is a necessary condition for observing shifting mortality. Is it a sufficient condition? This line of thought leads to a simple and easily tested hypothesis: if mortality change is proportional across age (the pattern of mortality change is similar across age on a relative scale), then there will be no change in the variability of age at death. Instead of becoming more or less variable, the death distribution will just parallel shift in either direction. If this relationship holds true, similarities in the relative pace of change in mortality across age might be the underlying cause of the current shifting mortality scenario.

In this paper, we seek to test this simple hypothesis: does proportional change in mortality that is fixed across age always lead to a shift in the death distribution. First, we explore the relationship between the age pattern of mortality change and variability of age at death analytically. We then go on to conduct simple experiments that test how measures of variability of age at death respond to changes in age-specific mortality rates under a variety of initial mortality conditions. Through perturbation analysis, we

[1] In the most recent period, the pattern is not similar across all ages but it is in the ages where the majority of deaths are concentrated. The average annual rate of mortality decline is calculated using the formula $-log\left(\frac{m_x(t+n)}{m_x(t)}\right)/n$.

are able to explore more directly the sensitivity of our measures of variability of age at death to changes in age-specific mortality without imposing experimental conditions. Our analysis leads us to a rejection of our simple hypothesis and a deeper understanding of the complex relationship between measures of variability of age at death, initial mortality conditions, and the age pattern of mortality decline.

2 Analytical understanding of shifting mortality

To begin, we want to consider the conditions under which a decline in mortality corresponds exactly to a shift in the mortality curve across age. A proportional decline in age-specific mortality rates corresponds to a shift of the mortality curve over age when the shape of the age-specific mortality curve is linear on a log scale. We often use the Gompertz curve, which fits this log-linear criteria, to describe the shape of the mortality curve at adult ages. This model can be expressed as follows:

$$\mu(x) = \alpha e^{\beta x}$$

A Gompertz mortality curve with $\alpha = 5 \times 10^{-5}$ and $\beta = .10$ is shown in Figure 3(a). It is clear from this figure that a proportional decline in age-specific log mortality rates that is fixed across age is equivalent to a shift of the mortality curve over some years of age. When mortality decline can be expressed as a shift of the mortality curve over age, the death distribution retains its shape yet shifts either forward or backward in age. This is shown in Figure 3(b).

While we are currently observing shifting death distributions akin to what is observed in this simple model of mortality change, current mortality conditions and changes in mortality over time do not necessarily conform to this simple model. First, the entire mortality curve over the whole age range is not well described by a Gompertz curve. As a result, a downward shift in mortality does not necessarily translate into a shift of the mortality curve over age (see Figure 4(a)). Additionally, proportional mortality change over time is not necessarily fixed across age. As shown in Figure 4(b), the age pattern of mortality change for Sweden has been divergent across age even in the period of time when relative stability in variance in life span has been observed (1960 onwards).

Our analysis of a simple model of mortality change has led us to consider the following discrepancies between the simple model and empirical reality: the divergence of the age pattern of mortality change and initial mortality conditions that are not well described by the Gompertz or logistic mortality models. The results of our analytical investigation suggest that our simple hypothesis might not hold. Proportional changes in mortality that are fixed across age will not necessarily translate into a shift in the death distribution if the initial mortality conditions are not linear or semi-linear on a log scale. In the next section, we explore further the dynamic relationship between initial mortality conditions and measures of variability of age at death given proportional change in mortality that is fixed across age and time.

3 Do proportional changes in mortality that are fixed across age result in shifting mortality?

We have set up a relatively simple experiment to test the hypothesis that a proportional change in mortality that is fixed across age will have no impact on variability of age at death. Starting with an initial set of age-specific mortality rates (m_x), we apply a proportional change in mortality that is fixed over age and time over some projection interval, and measure the variability of age at death over the projection horizon. The simple hypothesis would suggest that regardless of the initial set of age-specific mortality rates the variability of age at death should remain the same over the projection interval. The experiment is carried out using five different initial vectors of age-specific mortality rates, m_x: the single-year age-specific mortality rates of Swedish males in 1800-04, 1850-54, 1900-04, 1950-54, and 2000-04[2]. These initial mortality vectors come from data included in the Human Mortality Database [HMD, 2009]. We project the change in m_x over 300 years assuming an average annual proportional change $\phi = .005$ across all ages according to the following equation[3]:

$$m_x(t) = m_x(0) * (1 + \phi)^t \tag{3}$$

Variability of age at death is measured using both S_{10}, the standard deviation of ages at death past age 10, and $e^\dagger$, the years of life lost in the life table due to premature death: $e^\dagger = \int_0^w e_x d_x$. The $e^\dagger$ measure takes into account changes in infant mortality while the S_{10} measure will be more reflective of the mortality experience during adult ages. Actual trends in S_{10}, entropy ($H = \frac{e^\dagger}{e_0}$), and $e^\dagger$ for both deaths above age 10 and deaths at all ages for Swedish males and females are shown in Figure 5.

One difficulty with carrying out this experiment is that the oldest observable age is fixed at 110. As deaths start to accumulate in the open ended age interval, this can bias the results. With an annual proportional change of $\phi = .005$, there did not seem to be too many deaths accumulating in the last age group after projecting forward 300 years (for $\phi = .01$ there was very obvious bias in the measures of variability of age at death). Still, the trends in $e^\dagger$ and S_{10} near the end of the projection interval should be interpreted with caution.

Graphical results of this simple experiment are depicted in Figure 6. These results suggest that the relationship between the age pattern on mortality change and variability of age at death is not so straightforward as the simple hypothesis would suggest. When variability of age at death is measured as S_{10}, a proportional change in mortality results in relative stability in S_{10} over the projection interval for four of our five initial m_x distributions. For the initial set of age-specific mortality rates, m_x,

[2] We were also planning on using an initial m_x corresponding to the mortality rates of Swedish males in 1751-54, but the mortality rates at older ages for this life table are unusually low ($m_{85+}(1950) > m_{85+}(1751)$) which resulted in deaths heaping up too rapidly in the open ended age interval

[3] The following equation would have been more consistent with how Wilmoth and Horiuchi measured the average annual rate of proportional mortality decline.

$$m_x(t) = m_x(0)e^{\phi t} \tag{2}$$

corresponding to the mortality experience of Swedish males in 1900-04, a fixed proportional change in mortality across age and time results in a decline in S_{10} over the projection horizon. The divergence of the trend for the initial m_x corresponding to 1900-04 from the other initial states is quite striking, and it suggests that trends in the variability of age at death do not have a simple relationship with the age pattern of mortality decline. Rather, there is an interaction between the initial death distribution and the age pattern of mortality change. For some initial mortality conditions, like the conditions Swedish males experienced in 1900-04, measures of variability of age at death are more sensitive to changes in mortality-whether these changes are fixed across ages or not.

If instead of using S_{10} as the measure of variability of age at death we use $e^\dagger$, a different picture of mortality change emerges. In this case, the trends in variability of age at death still look relatively stable for the initial m_x's corresponding to 1950-54 and 2000-04; however, the initial m_x's corresponding to 1800-04 and 1850-54 show a declining trend in $e^\dagger$ similar to the trend observed with initial m_x's corresponding to 1900-04 except less steep. The difference between trends observed for $e^\dagger$ and S_{10} corresponding to initial m_x vectors from 1800-04 and 1850-54 is the result of the responsiveness of $e^\dagger$ to changes in infant mortality. As will be demonstrated later, $e^\dagger$ is particularly sensitive to changes in infant mortality especially when the initial states indicate high levels of infant mortality.

The results of this simple experiment of applying a fixed proportional change in mortality over age and time to several initial sets of age-specific mortality rates overwhelmingly demonstrate that our initial hypothesis was not true. An age pattern of proportional mortality change that is similar across ages does not in and of itself lead to stability in the measure of variability of age at death. Proportional change in mortality that is fixed across age while perhaps a necessary condition is certainly not a sufficient condition for shifting mortality.

4 Further examination of the relationship between age pattern of mortality change and variability of age at death

In order to gain more insight into the relationship between the age pattern of mortality change and initial mortality conditions, we conducted a similar experiment but used patterns of mortality change that varied over age while remaining fixed over time. Two different patterns of age-specific mortality change were utilized: the age patterns of average annual proportional mortality change observed during the periods 1875-79 to 1950-54 and 1950-54 to 1990-94 among Swedish males. The data on mortality change, like the set of initial mortality vectors, also come from the Human Mortality Database [HMD, 2009]. The underlying age patterns of average annual mortality change along with the smoothed age pattern utilized in the analysis are shown in Figure 7. The projections in this experiment only extend out 100 years because further extending the time horizon resulted in too many deaths being heaped into the open ended age group.

Wilmoth and Horiuchi observed in their paper that the age pattern of mortality

change was much more divergent across ages during the period 1875-79 to 1950-54 when the IQR was declining rapidly in comparison to the the period 1950-54 to 1990-94 when the age pattern is much flatter across the ages where the majority of deaths are concentrated [Wilmoth and Horiuchi, 1999]. Therefore, we expect that applying the age pattern of mortality change observed in 1875-79 to 1950-54 to the initial m_x vectors will result in faster mortality compression (reduction in variability of age at death) than when we use the average annual proportional change observed from 1950-54 to 1990-94 to make our projections. We are especially interested in observing how rates of decline in S_{10} and $e^{\dagger}$ differ depending on the initial m_x distribution.

The results of this analysis are shown in Figure 6. As anticipated, applying the average annual mortality change observed in the period 1875-79 to 1950-54 to the five initial m_x vectors resulted in faster mortality compression than using the mortality change observed 1950-54 to 1990-94. When variability of age at death is measured using S_{10}, the fastest rates of mortality compression using either pattern of mortality change are observed when the initial m_x corresponds to the m_x observed for Swedish males 1900-04. Again, it appears that this initial distribution is more "primed for compression" in comparison to the other initial sets of m_x. If $e^{\dagger}$ is employed to measure variability instead of using S_{10}, the initial set of m_x's corresponding to 1800-04, 1850-54, and 1900-04 exhibit similar rates of mortality compression although the set of m_x's observed in 1800-04 exhibits slightly faster compression near the end of the projection interval, but we should be careful when interpreting trends at the end of the projection interval because of death heaping in the open ended age interval.

The contrast between the rates of decline in both S_{10} and $e^{\dagger}$ for the initial m_x's corresponding to 1950-54 and 2000-04 in comparison to the initial m_x's corresponding to 1800-04, 1850-54, and 1900-04 is noteworthy. Applying the same proportional change in mortality over time, results in much less mortality compression if the initial mortality state corresponds to 1950-54 or 2000-04 in comparison to earlier eras. Nonetheless, especially for the projection with the age pattern of mortality change corresponding to what was observed 1875-79 to 1950-54, the variability of age at death does decrease over the projection interval when m_x's corresponding to 1950-54 and 2000-04 are used as starting states. Therefore, the shifting mortality phenomenon currently being observed in industrialized countries is not due solely to having reached some sort of threshold level of mortality beyond which the variability of age at death is free of influence from the age pattern of mortality change. As will be demonstrated later, though, the ages at which mortality change matters most for these measures of variability of age at death have become increasingly compressed.

This simple analysis of applying fixed rates of proportional mortality change over time has yielded a number of insights. Greater divergence in the age pattern of mortality decline results in faster mortality compression than age patterns that are similar using a number of initial mortality conditions. Also, the rate of decline in variability of age at death given a fixed change in mortality over time depends on the initial mortality conditions. Certain initial mortality states are "primed for compression." We investigate this further in the next section using perturbation analysis.

5 Sensitivity of S_{10} and $e^\dagger$ to change in mortality

Through our experiments projecting mortality rates forward using an age pattern of mortality change that is fixed over time, we have discovered that the pace of mortality compression depends on the initial mortality state. Using perturbation analysis, we are able to quantify the sensitivity of measures of variability of age at death to changes in age-specific mortality. In this section, we will calculate the sensitivities of S_{10} and $e^\dagger$ to changes in age-specific mortality rates using the initial set of m_x vectors that were used in the analysis in the previous section. We anticipate that the measures of variability of age at death will appear particular sensitive to changes in mortality for the set of m_x's corresponding to the mortality conditions observed for Swedish males in 1900-04. We calculate the sensitivity of S_{10} and $e^\dagger$ to changes in age-specific mortality through perturbation analysis of a Markov chain model. These perturbation analysis methods, currently being developed by Hal Caswell, are extensions of the methods found in [Caswell, 2006, Caswell, 2009, in press].[4]

The sensitivity of S_{10} to changes in age-specific mortality are presented in Figure 8. The graphs indicate the response of S_{10} to either absolute or proportional *increases* in mortality at each age. S_{10} grows more sensitive to absolute changes in mortality at younger ages over time as mortality rates at younger ages becomes increasingly low. The bottom graph, which shows the absolute response of S_{10} to proportional changes in mortality at a particular age is perhaps more interesting. As anticipated, when S_{10} is estimated from the set of m_x's corresponding to mortality in 1900-04 for Swedish males, the measure exhibits greater sensitivity to changes in mortality in the age groups 15-40 in comparison to S_{10} estimated from the set of m_x's corresponding to 1800-04 and 1850-54. This suggests that rapid decreases observed in variability of age at death from 1876-1955 were attributable not only to divergence in the age pattern of mortality but also partly to the greater sensitivity of measures of variability of age at death to changes in age-specific mortality due to the initial mortality conditions.

This view is reinforced by looking at the sensitivity of $e^\dagger$ to changes in age-specific mortality rates. As shown in Figure 9, the measure $e^\dagger$ exhibits much more sensitivity to changes in mortality in infancy, childhood, and young adult ages for the initial set of m_x's corresponding to 1900-04 in comparison to the other initial sets of m_x's. For the initial m_x's corresponding to 1800-04 and 1850-54, $e^\dagger$ is not very sensitive to mortality declines from roughly age 10 to age 30 and after some point close to age 40 declines in mortality would actually result in an increase in $e^\dagger$ (Zhang and Vaupel refer to this crossover age as the "age separating early from late deaths" [Zhang and Vaupel, 2009]). In contrast, declines in mortality up to almost age 60 will result in a decrease in $e^\dagger$ and $e^\dagger$ will respond more readily to the same proportional change in mortality at younger ages if the initial mortality conditions correspond to the set of m_x's from 1900-04. For the initial sets of mortality rates corresponding to 1950-54 and 2000-04, declines in mortality into even older ages would result in a decline in $e^\dagger$, but $e^\dagger$ is much less sensitive changes in mortality in infancy, childhood, and early adulthood in comparison to the set of m_x's corresponding to 1900-04.

The sensitivity of $e^\dagger$ to changes in infant mortality for Sweden during the period

[4]The perturbation analysis methods were presented in Hal Caswell's "Perturbation Analysis of Longevity" course offered March 30-April 17, 2009 at the IMPRSD in Rostock, Germany.

1751-2004 is shown in Figure 10. As can be seen in this graph, if we consider absolute changes in m_0, $e^\dagger$ became more sensitive to changes in infant mortality over time as a result of increasing life expectancy over this period. If we consider proportional changes in mortality at age 0, the sensitivity of $e^\dagger$ to these changes peaks around 1900. The decrease in the sensitivity of $e^\dagger$ and S_{10} to changes in infant mortality after this point can be attributed to the decline in infant mortality rates.

As mortality conditions improve over time (from 1800-04 to 2000-04) $e^\dagger$ becomes less responsive to changes in infant and childhood mortality while both $e^\dagger$ and S_{10} become less responsive to changes in mortality during early adulthood. As a result of the decreasing influence of early life mortality, the shape of the sensitivity curve changes in a very fundamental way. If we ignore the "accident hump" around age 20, there is a general trend towards a concentration of the ages where S_{10} and $e^\dagger$ are most sensitive to changes in mortality. When we consider just the ages where increases in mortality lead to an increase in variability of age at death, for both $e^\dagger$ and S_{10}, the measures actually become more sensitive to changes in mortality at older age ages in comparison to younger when the initial m_x's correspond to 1950-54 or 2000-04. In contrast, for the initial m_x's correspond to 1800-04, 1850-54, and 1900-04, changes in mortality at younger ages are always more important for S_{10} and $e^\dagger$ than changes near the cross-over age (the age where sensitivity of $e^\dagger$ or S_{10} to changes in mortality is zero). The ages where an increase in mortality leads to decrease in S_{10} become increasingly compressed over time as mortality conditions improve. Mortality conditions in 1900 were "primed for compression" not only because $e^\dagger$ and S_{10} were more responsive to changes in mortality at younger ages in comparison to other initial m_x's, but also because the ages where declines in mortality would lead to increase in S_{10} were more compressed than for the set of m_x's corresponding to 1800-04 and 1850-54.

In the next section, we examine actual trends in S_{10} for Sweden in the 19th and 20th centuries. Through decomposition analysis, we are able to quantify the contribution of changes in age-specific mortality rates to changes in S_{10} over time. These age-specific contributions reflect both the potential of mortality change at that age to impact S_{10} as well as the pace of mortality decline at that age relative to other age groups. While these decomposition results are reflective of the actual age patterns of mortality change over time, which might be divergent or more similar across age, the results are nonetheless very closely related to the results of the sensitivity analysis that only reflect what would occur if proportional mortality change was fixed across age.

6 Decomposition of temporal trends in measures of variability of age at death

Throughout this paper, we have shown that changes in measures of variability of age at death are influenced both by the initial mortality conditions as well as the age pattern of mortality change. As discussed in the last section, the age pattern of sensitivity of S_{10} and $e^\dagger$ to changes in mortality has changed fundamentally in Sweden over the course of the 19th and 20th centuries. In this section, we explore how actual age-specific mortality change has influenced actual trends in variability of age at death in Sweden during the late 19th century and throughout the 20th century. Using decomposition

techniques, we are able to quantify the contribution of changes in age-specific mortality to changes in S_{10} over this time period. Trends in S_{10} are derived separately by sex using data from life tables included in the HMD with one-year age groups that cover five-year periods [HMD, 2009].

The results of our sensitivity analysis indicate that we should expect the age pattern of contributions of changes in mortality to changes in S_{10} to be modified over the course of the period of observation. We expect that mortality change at younger ages will be most important in determining the changes in the variability of age at death in the earlier part of our period of observation. Changes at older ages will become increasingly important in the latter part of the period. We expect this both because the age pattern of mortality change is more rapid at younger ages in comparison to older ages in the first part of the twentieth century (see Figure 4(b)) and because S_{10} is more sensitive to changes in mortality at younger ages during this earlier period (see Figure 8).

We decompose trends in S_{10} by age using a method proposed by Horiuchi et al. [Horiuchi et al., 2008]. We look specifically at the contributions of changes in age-specific mortality rates, m_x, to overall changes in S_{10} over time. More details on this decomposition method are provided in the appendix. We carry out the decompositions for Swedish males and females over three time periods: 1880-84 to 1920-24, 1920-24 to 1960-64, and 1960-64 to 2000-04. During both the first and third time periods, trends in S_{10} are relatively stable while a large decrease in S_{10} is observed between 1920-24 to 1960-64 (see Figure 5).

The results of this decomposition analysis are presented in Figure 11 and in Table 1. As expected, the rapid decline in S_{10} observed during the period 1920-24 to 1960-64 was due to mortality changes at younger ages with the peak contribution around age 20. In the most recent period, 1960-64 to 2000-04, the contributions of changes in mortality at younger ages, which would decrease S_{10} further, are increasingly being matched by mortality improvements at older ages, which act to increase S_{10}. As indicated by the sensitivity analysis, in more recent periods, the ages at which mortality change matters most for trends in the variability of death are becoming increasingly compressed.

Unlike the age pattern of sensitivity of S_{10} corresponding to initial mortality conditions observed in Sweden during the latter half of the 20th century, the age pattern of contributions of changes in age-specific mortality rates to changes in S_{10} over the period 1960-64 to 2000-04 only has one hump at the oldest ages. The age pattern of sensitivity curve for 1950 and 2000, in contrast, contains two humps on either side of the cross-over age. The lack of a hump before the cross-over age in the decomposition results might be due divergence in the age pattern of mortality change at younger ages. The decomposition results reflect the cumulative effect of both the sensitivity of the initial mortality conditions and divergence in the age pattern of mortality.

7 Conclusion

This analysis has revealed that both the age pattern of mortality change and current mortality conditions are important in determining the future course of variability of age at death. The rapid declines in variability of age at death observed in Sweden from 1876-1955 were due to both divergence in the age pattern of mortality decline

and the greater sensitivity of measures of variability of age at death to changes in age-specific mortality rates during this time period. Similarly, the current stability in S_{10} is likely due partly to the similarity in rates of mortality change across age as well as the compression of the age interval where measures of variability of age at death are most sensitive to changes in mortality.

Given the changes over the course of the 20th century in the age pattern of sensitivity of measures of variability of age at death to changes in mortality, it is unlikely that we will observe further rapid declines in life span variability in countries like Sweden. As the ages where mortality change matters most for trends in variability become increasingly compressed, a pattern of mortality change that is divergent across age becomes less important.

Still, it is not clear that the age pattern of mortality change should be completely disregarded as we contemplate future mortality change. While it is tempting to adopt new forecasting models that take advantage of the shifting conditions currently observed, these mortality shifts may not necessarily be sustained in the long term if the age pattern of mortality change becomes more divergent across age in the future. Future research on the possible transition to shifting mortality should focus not only on trends in variability of age at death but also on likely trends the age pattern of mortality change.

References

[HMD, 2009] (2009). Human Mortality Database. University of California, Berkeley (USA) and Max Planck Institute for Demographic Research (MPIDR). Available at www.mortality.org (data downloaded on February 4, 2009).

[Bongaarts, 2005] Bongaarts, J. (Feb., 2005). Long-range trends in adult mortality: Models and projection methods. *Demography*, 42(1):23–49.

[Canudas-Romo, 2008] Canudas-Romo, V. (2008). The modal age at death and the shifting mortality hypothesis. *Demographic Research*, 19(30):1179–1204.

[Caswell, 2006] Caswell, H. (2006). *MAM2006: Markov Anniversary Meeting*, chapter Applications of Markov chains in demography, pages 319–334. Boson Books, Raleigh, North Carolina, USA.

[Caswell, 2009] Caswell, H. (2009). Stage, age and individual stochasticity in demography. *Oikos. in press.*

[Cheung and Robine, 2007] Cheung, S. L. K. and Robine, J.-M. (2007). Increase in common longevity and the compression of mortality: the case of Japan. *Popul Stud (Camb)*, 61(1):85–97.

[Cheung et al., 2005] Cheung, S. L. K., Robine, J.-M., Tu, E. J.-C., and Caselli, G. (2005). Three dimensions of the survival curve: horizontalization, verticalization, and longevity extension. *Demography*, 42(2):243–258.

[Edwards and Tuljapurkar, 2005] Edwards, R. D. and Tuljapurkar, S. (2005). Inequality in life spans and a new perspective on mortality convergence across industrialized countries. *Population and Development Review*, 31(4):645–674.

[Fries, 1980] Fries, J. F. (1980). Aging, natural death, and the compression of morbidity. *N Engl J Med*, 303(3):130–135.

[Fries, 1984] Fries, J. F. (1984). The compression of morbidity: miscellaneous comments about a theme. *Gerontologist*, 24(4):354–359.

[Horiuchi et al., 2008] Horiuchi, S., Wilmoth, J. R., and Pletcher, S. (2008). A decomposition method based on a model of continuous change. *Demography*, 45(4):785–801.

[Kannisto, 1996] Kannisto, V. (1996). *The advancing frontier of survival: life tables for old age*. Odense University Press, Odense.

[Kannisto, 2000] Kannisto, V. (2000). Measuring the compression of mortality. *Demogr Res*, 3:[24] p.

[Kannisto, 2001] Kannisto, V. (2001). Mode and dispersion of the length of life. *Population: An English Selection*, 13(1):159–171.

[Myers and Manton, 1984a] Myers, G. C. and Manton, K. G. (1984a). Compression of mortality: myth or reality? *Gerontologist*, 24(4):346–353.

[Myers and Manton, 1984b] Myers, G. C. and Manton, K. G. (1984b). Recent changes in the U.S. age at death distribution: further observations. *Gerontologist*, 24(6):572–575.

[Robine, 2001] Robine, J.-M. (2001). Redefining the stages of the epidemiological transition by a study of the dispersion of life spans: The case of France. *Population: An English Selection*, 13(1):173–193.

[Robine, 2008] Robine, J.-M. (2008). Between compression and shifting mortality the longevity revolution. In *European Papers on the New Welfare*, number 9, The Turin Conference on the New Welfare. The Risk Institute.

[Thatcher et al., 2008] Thatcher, A. R., Cheung, S. L. K., Horiuchi, S., and Robine, J.-M. (2008). The compression of deaths above the mode. Population Association of America.

[Wilmoth and Horiuchi, 1999] Wilmoth, J. R. and Horiuchi, S. (1999). Rectangularization revisited: variability of age at death within human populations. *Demography*, 36(4):475–495.

[Zhang and Vaupel, 2009] Zhang, Z. and Vaupel, J. (2009). The age separating early deaths from late deaths. *Demographic Research*, 20(29):721–730.

A Decomposition method

We are interested in decomposing trends in S_{10} over time into the contributions of changes in age-specific mortality rates. We carry out these decompositions using the continuous-change method of decomposition developed by Horiuchi et al. This is a general method of decomposition that relies upon the assumptions that the covariates whose effects are being measured (in our case the underlying m_x's) change gradually over time and that covariates change proportionally to one another over time. If these two assumptions are met, the decomposition method allows us to estimate the contribution, c_i, of changes in a particular covariate to changes in the measure of interest. For a more detailed description of this decomposition method see [Horiuchi et al., 2008]. In our case, we are interested in determining the contribution of changes in m_x to the total change in S_{10}.

As a first step to carrying out this decomposition, we must find an expression for S_{10} in terms of m_x.

$$S_{10} = \sqrt{\frac{\sum_{x=10}^{T}(x + a_x - M_{10})^2 \times d_x}{\sum_{x=10}^{T} d_x}}$$

where M_{10} is $e_{10} + 10$ and T is the oldest age group

$$S_{10} = \sqrt{\frac{\sum_{x=10}^{T}(x + a_x - M_{10})^2 \times l_x m_x}{\sum_{x=10}^{T} l_x m_x}}$$

$$S_{10} = \sqrt{\frac{\sum_{x=10}^{T}(x + a_x - \sum_{x=10}^{T} x e^{\sum_0^x -m_x} m_x)^2 \times e^{\sum_0^x -m_x} m_x}{\sum_{x=10}^{T} e^{\sum_0^x -m_x} m_x}}$$

We are interested in attributing the changes in S_{10} over time to the changes in m_x over the same time period. Therefore, it is important to recognize that S_{10} and m_x are functions of time:

$$S_{10}(t) = \sqrt{\frac{\sum_{x=10}^{T}\left(x + a_x - \sum_{x=10}^{T} x e^{\sum_0^x -m_x(t)} m_x(t)\right)^2 \times e^{\sum_0^x -m_x(t)} m_x(t)}{\sum_{x=10}^{T} e^{\sum_0^x -m_x(t)} m_x(t)}}$$

To use the Horiuchi et al. method we note the following [Horiuchi et al., 2008]:

$$S_{10} = f(m_1, m_2, ..., m_T)$$

The general decomposition method holds that for some variable, y, where $y = f(x_1, x_2, ..., x_n)$, the change in y from time 1 to time 2 can be expressed as:

$$y_2 - y_1 = \sum_{i=1}^{n} c_i, \text{ where } c_i = \int_{X_{i1}}^{X_{i2}} \frac{\partial y}{\partial x_i} dx_i$$

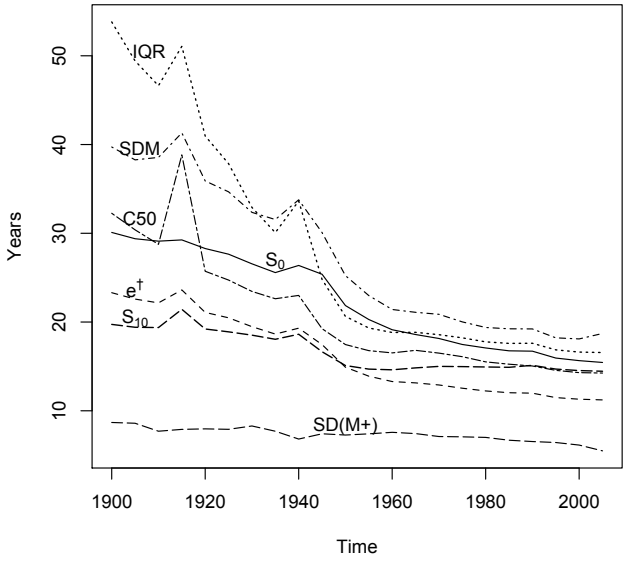
The equation above indicates that the total change in the variable of interest, y, from time 1 to time 2 can be found by summing up the contributions, c_i of each covariate, x_i. Computationally, estimating c_i involves looking at how changes in a particular value of x_i over a narrow interval change y while holding other values of x_i constant at values observed at the midpoint of the narrow interval and then summing the effects observed across all of the intervals between time 1 and 2.

For our analysis, we want to take partial derivatives of S_{10} with respect to m_i to find the contribution of changes in the age-specific mortality rate, m_i, to changes in S_{10}:

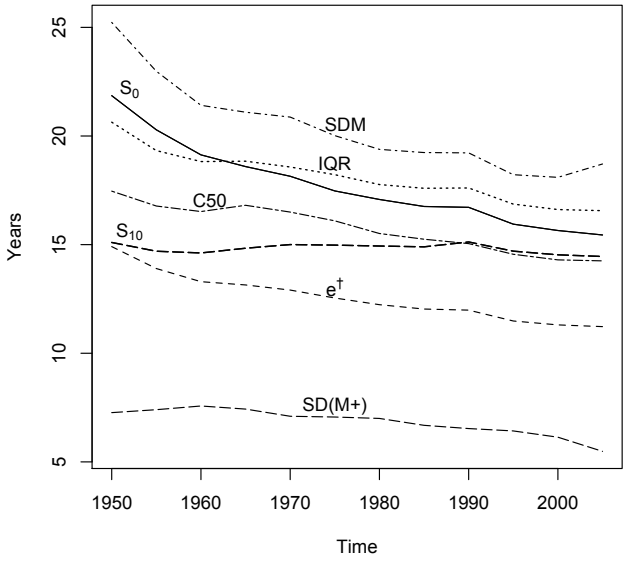
$$c_i = \int_{m_{i1}}^{m_{i2}} \frac{\partial S_{10}}{\partial m_i} \frac{dm_i}{dt}$$

Table 1: Decomposition of trends in S_{10} into the contributions of changes in age-specific mortality rates, Sweden, 1880-2004.

	Male			Female		
Age group	1880-84 to 1920-24	1920-24 to 1960-64	1960-64 to 2000-04	1880-84 to 1920-24	1920-24 to 1960-64	1960-64 to 2000-04
10-29	-1.39	-3.87	-0.77	-1.45	-4.53	-0.56
30-49	-0.48	-1.57	-0.74	-0.48	-2.21	-0.8
50-69	0.44	-0.1	-0.45	0.28	-0.39	-0.67
70+	0.58	0.21	1.1	0.39	0.51	1.52
Total	-0.85	-5.33	-0.86	-1.27	-6.62	-0.51



(a) 1900-2006



(b) 1950-2006

Figure 1: Trends in a variety of measures of mortality compression, France.

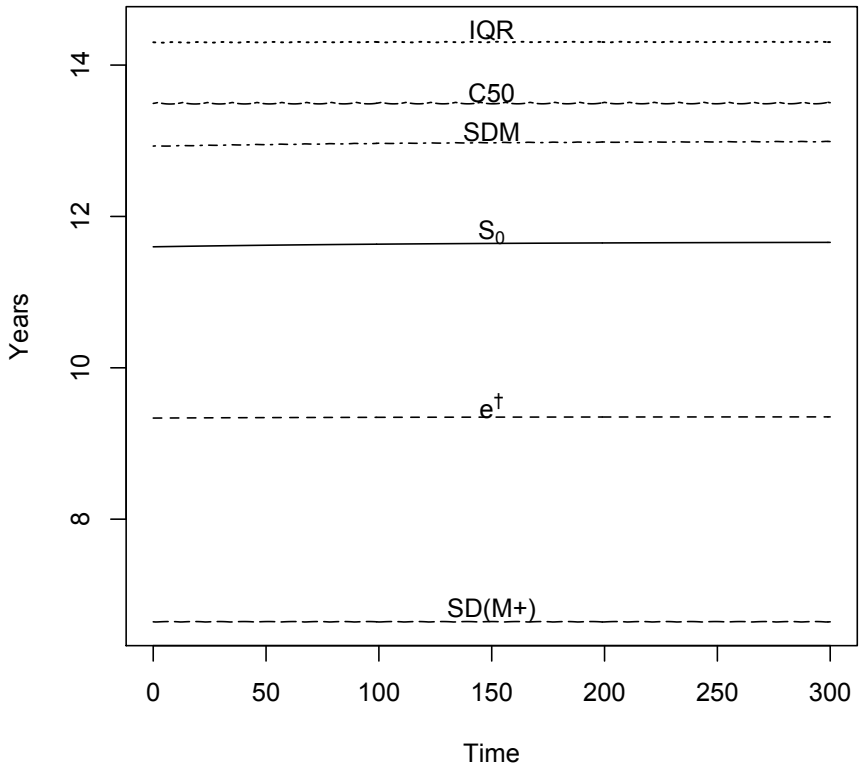
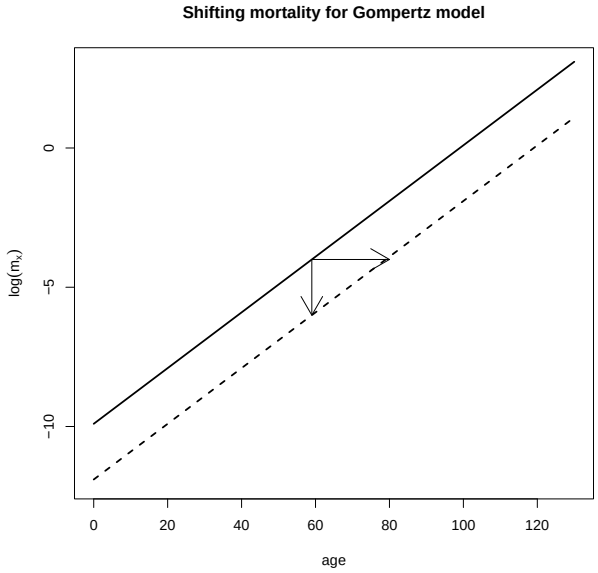
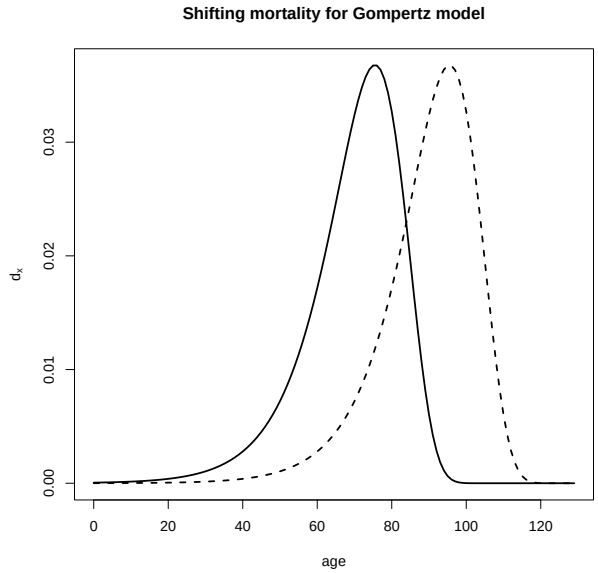


Figure 2: Trends in measures of mortality compression observed under the Gompertz mortality change model, $m_x = e^{\alpha+\beta x-\rho t}$, with $\alpha = -10.5$, $\beta = 0.11$, and $\rho = 0.01$. These initial conditions correspond to those found in [Canudas-Romo, 2008].

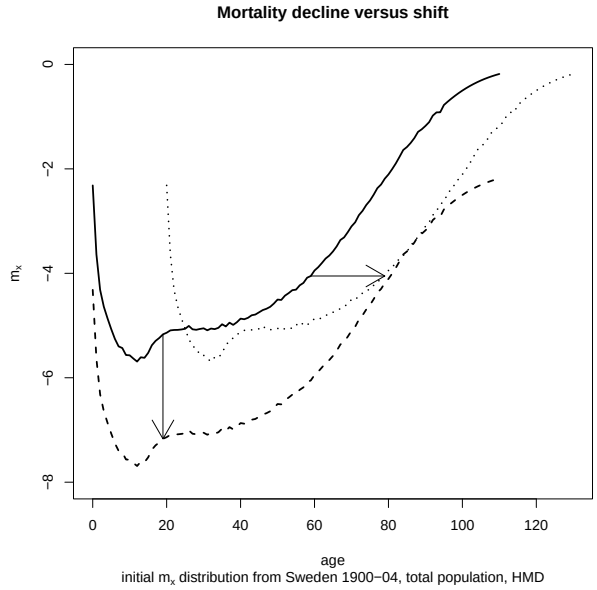


(a) Changes in age-specific mortality rates under the Gompertz model with proportional change in mortality across age

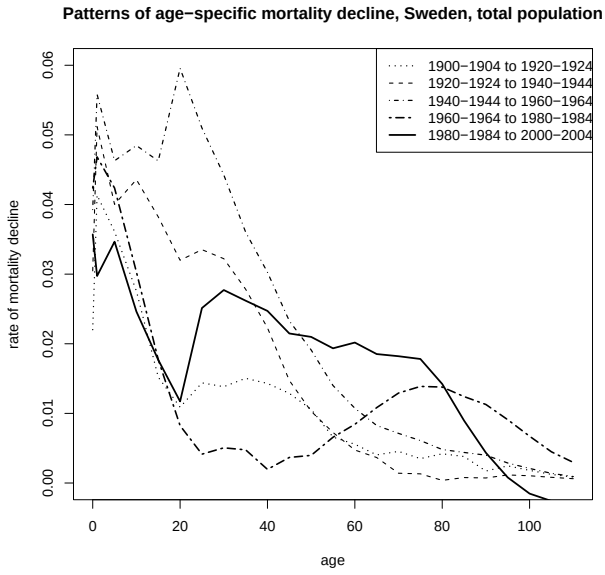


(b) Changes in the death distribution under the Gompertz model with proportional change in mortality across age

Figure 3: Shifting mortality under the Gompertz mortality change model.



(a) A decline in mortality does not necessarily translate into a shift in mortality



(b) The age pattern of mortality decline is divergent across age

Figure 4: Violations of the Gompertz mortality change model found in the mortality experience of actual human populations.

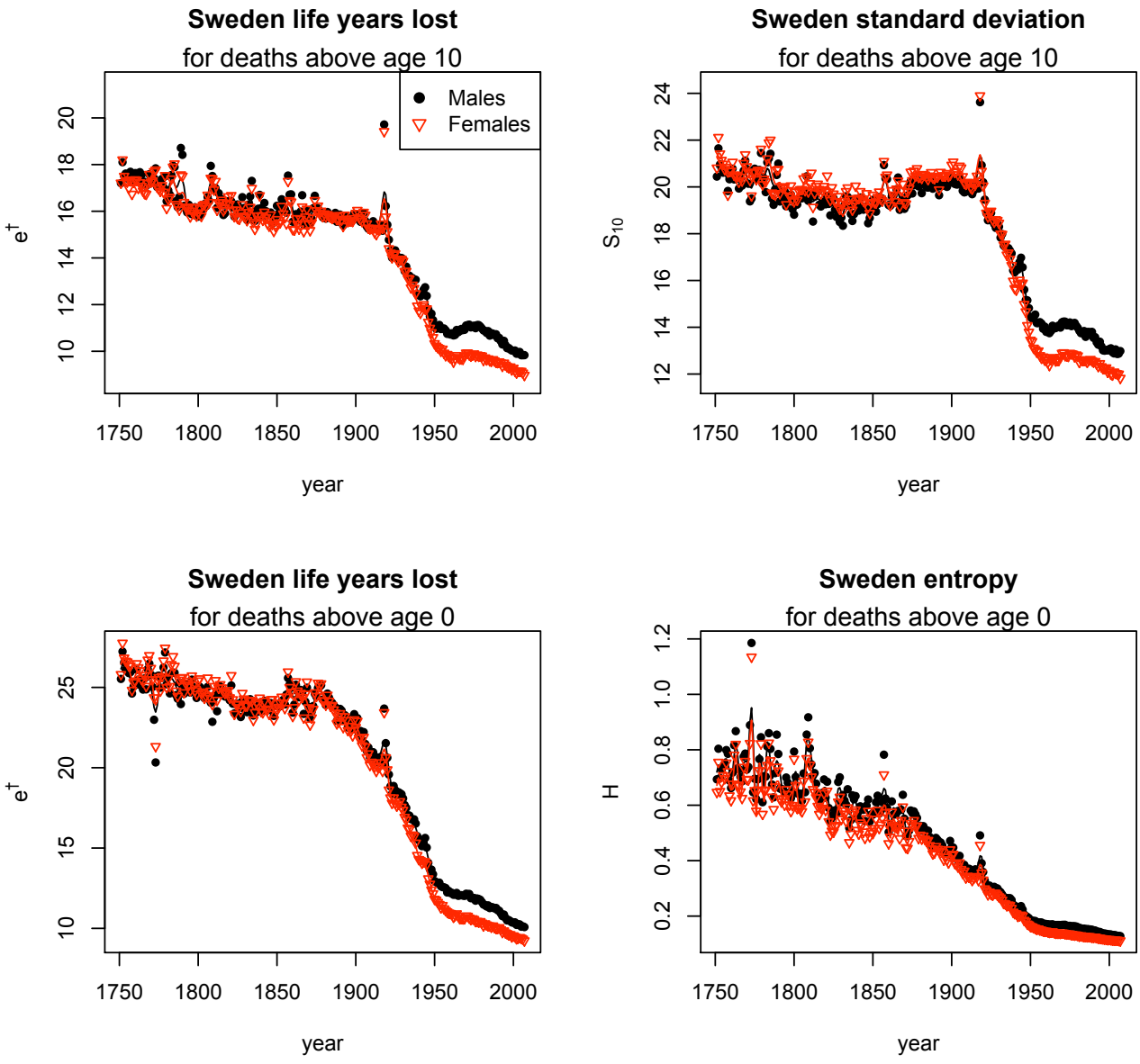


Figure 5: Trends observed in S_{10}, H, and $e^{\dagger}$, Sweden, 1751-2007

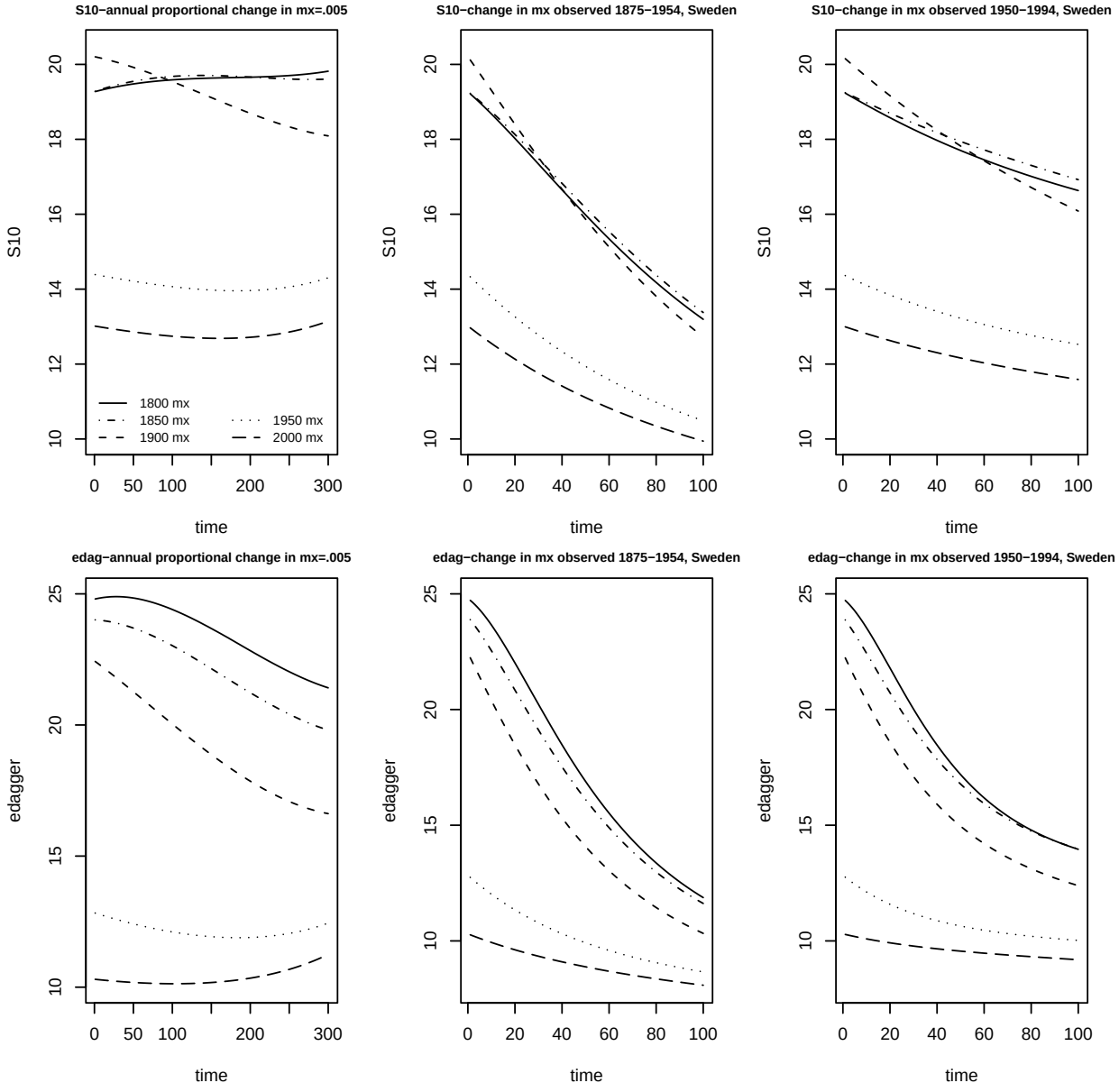


Figure 6: Trends observed in S_{10} and $e^{\dagger}$ under various projection scenarios

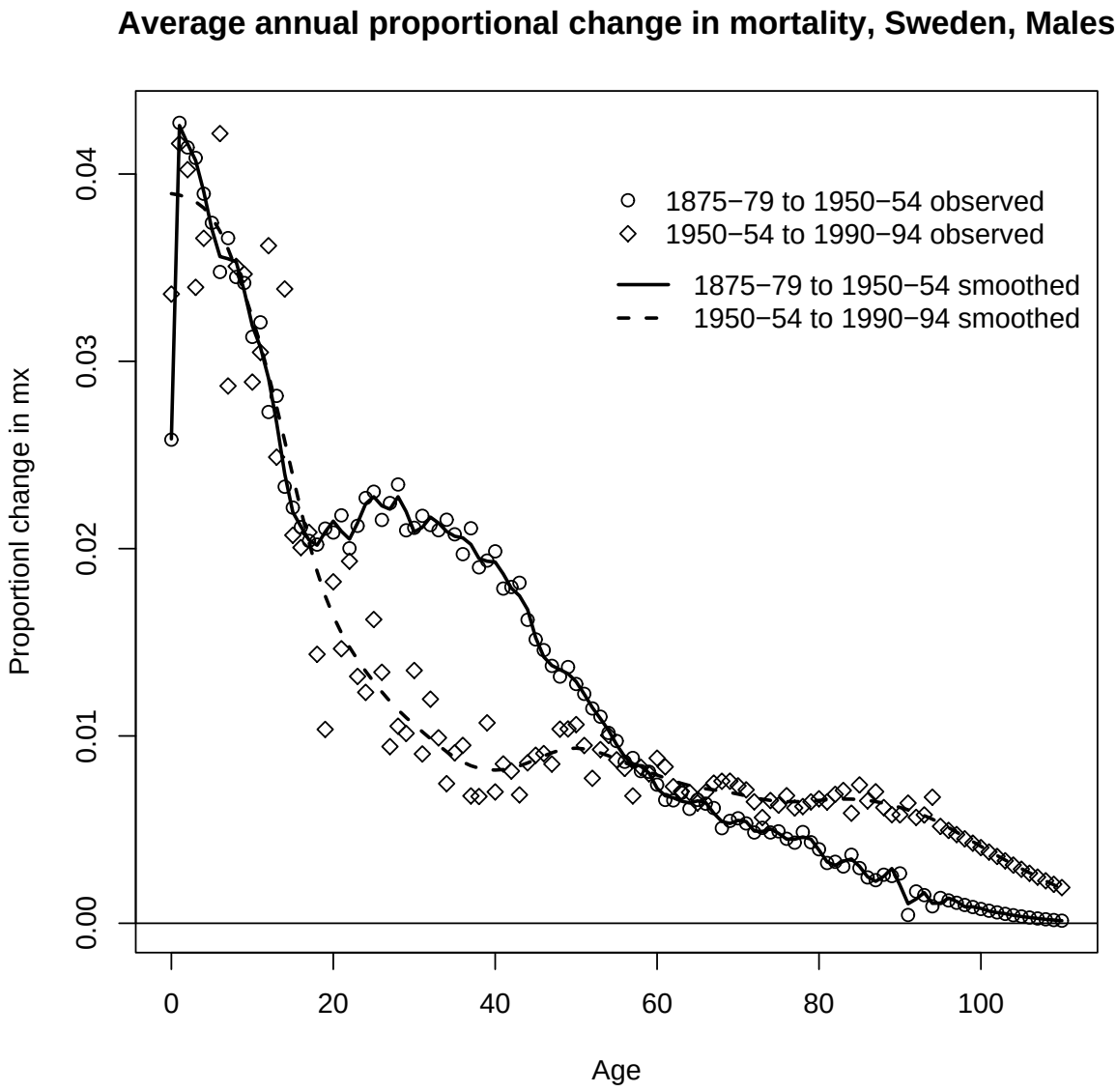


Figure 7: Observed and smoothed average annual proportional change in mortality for Swedish males 1875-79 to 1950-54 and 1950-54 to 1990-94

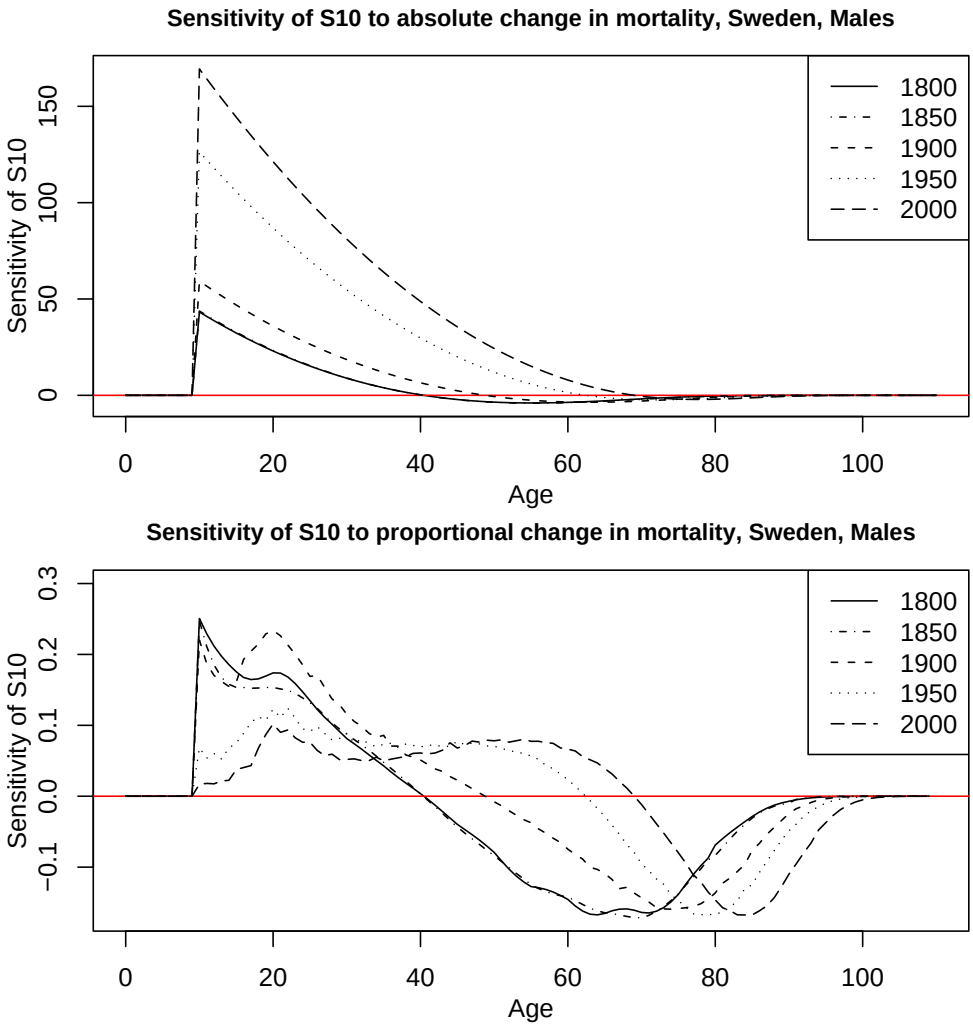


Figure 8: Sensitivity of S_{10} to either absolute or proportional changes in mortality at different ages

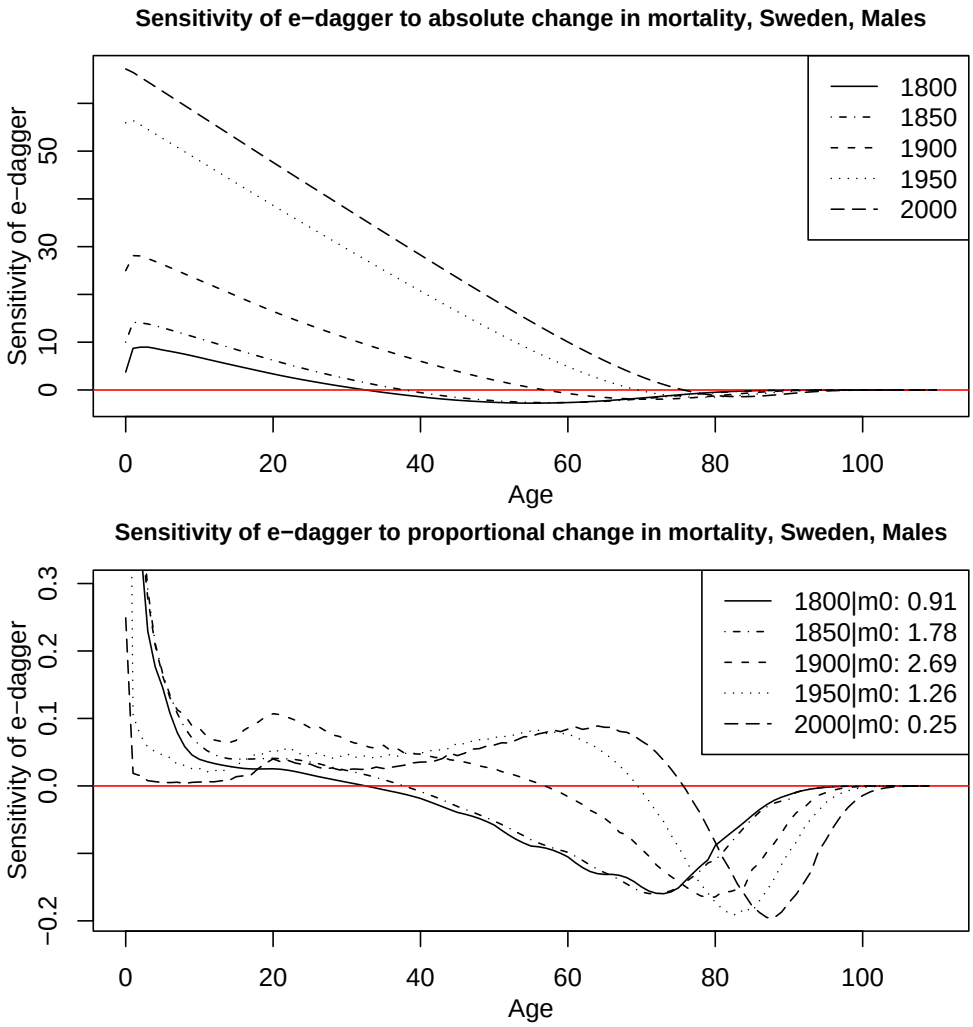


Figure 9: Sensitivity of $e^{\dagger}$ to either absolute or proportional changes in mortality at different ages

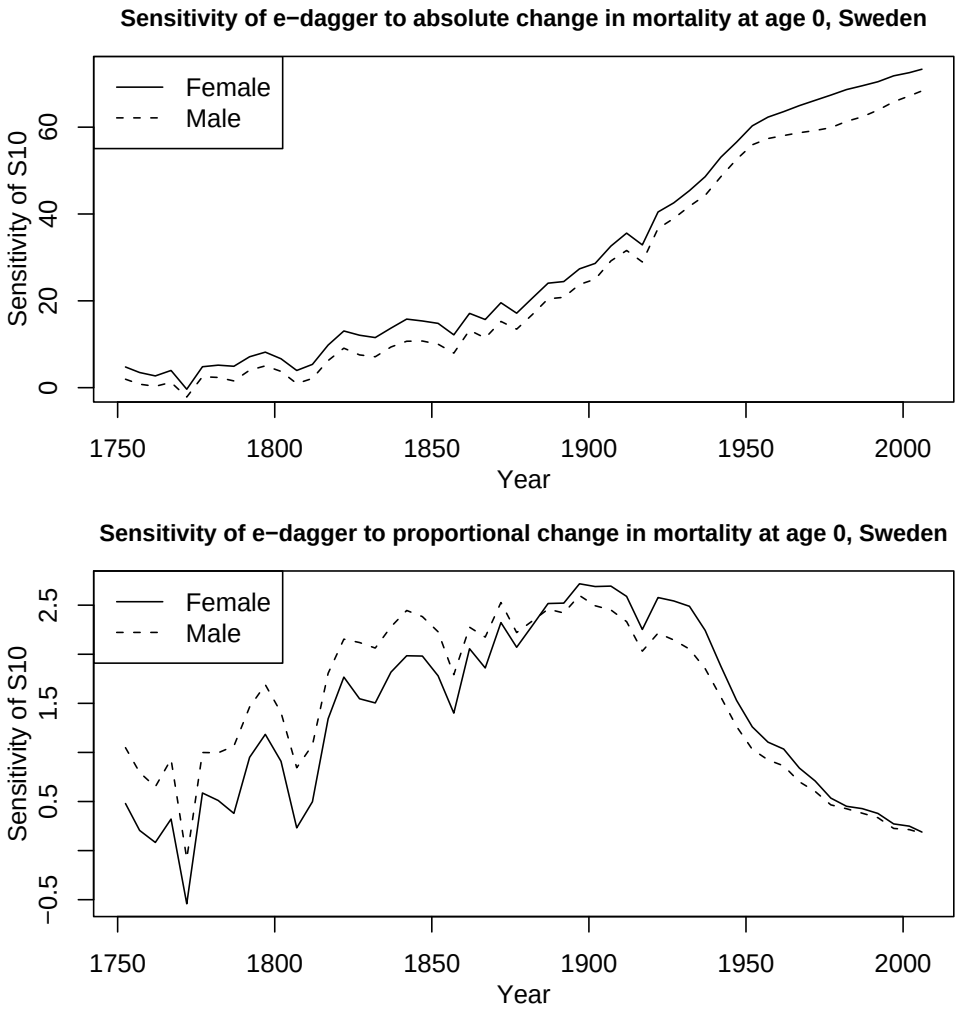


Figure 10: Sensitivity of $e^{\dagger}$ to either absolute or proportional changes in mortality at age 0 for Swedish males

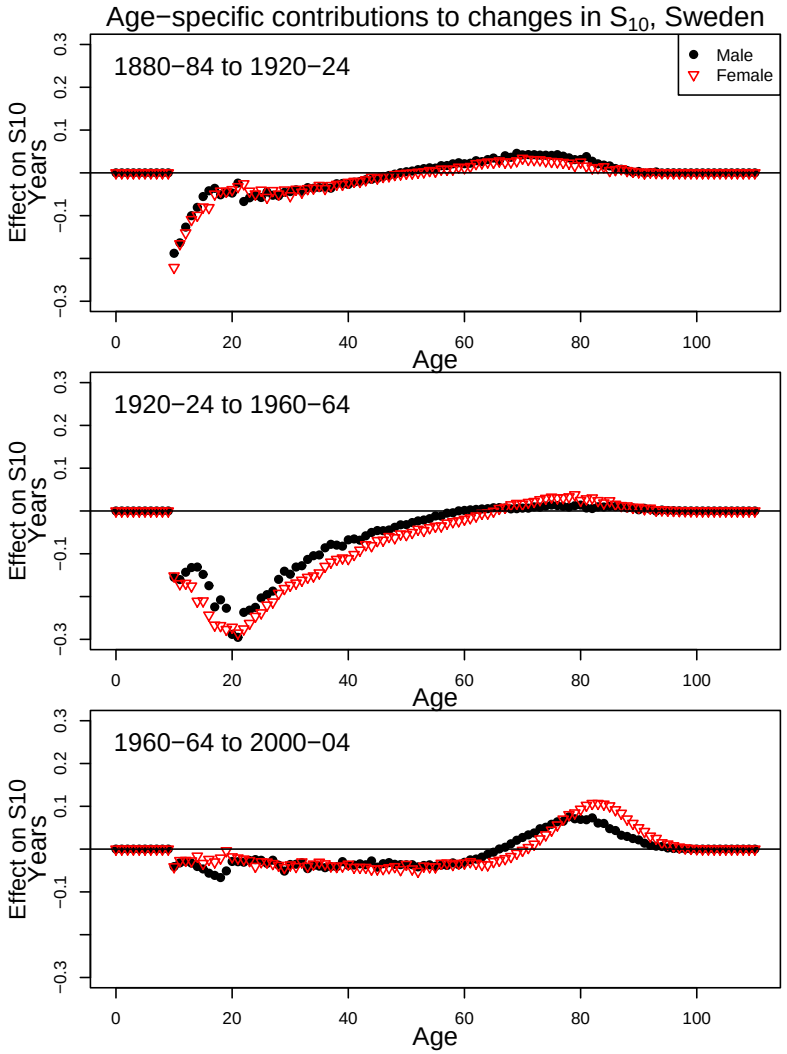


Figure 11: Results of decomposition of trends in S_{10} into the contributions of changes in age-specific mortality rates, Sweden, 1880-2004