

A Unifying Framework for Assessing Changes in Life Expectancy Associated with Changes in Mortality: The Case of Violent Deaths*

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Abstract

Policymakers often require an assessment of possible gains in life expectancy that could result from a large-scale public health campaign aimed at reducing mortality for specific ages and causes of death. Equally important is assessing the contribution of observed decline in a particular cause of death on observed gains in life expectancy. For over forty years, demographers have worked intensively to develop methods that address these important issues. As yet, there has been no framework unifying these important works. In this paper, we provide a unifying framework for assessing the change in life expectancy given any conceivable change in age and cause-specific mortality. We consider both conceptualizations of mortality change—counterfactual assessment of a hypothetical change and a retrospective assessment of an observed change. We apply our methodology to violent deaths, the leading cause of death among young adults, and show that realistic targeted reductions could have important impacts on life expectancy.

Key Words: Accidents, Cause-Deleted Life Tables, Decomposition, Entropy, Violent Deaths

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1 Introduction

Before engaging in large-scale and costly public health campaign, policymakers must know its potential impact on the population (Shelton et al., 2001; Medford and Oesch, 2009). For example, how much gain in life expectancy could one expect if mortality declined in a particular manner? Equally important is assessing a campaign's impact on key outcomes. How much of an observed gain in life expectancy may be attributed to the observed decline in a particular cause of death? The answers to these counterfactually and causally framed questions, of course, depend on the magnitude and ages of mortality decline.

For over forty years, demographers have worked intensely to develop methods that assess the gain in life expectancy from a reduction in mortality, either hypothetical or observed. The first attempt came in the form of single decrement life tables which estimated the gain in life expectancy at birth under the assumption that one cause of death was completely eliminated (United States Department of Health, Education, and Welfare, 1968). Recognizing the tenuousness of this assumption, Keyfitz (1977) derived the proportional change in life expectancy at birth when either all-cause or cause-specific mortality was reduced by a constant percentage across age. Later developments focused on decomposition approaches. For example, Pollard (1982), and Arriaga (1982) examined absolute gains in life expectancy resulting from absolute reductions in cause and age specific mortality between two discrete times. Subsequently, Vaupel (1986) and Vaupel and Canudas Romo (2003) further developed decomposition approaches from a continuous-time perspective focusing on continuous progress against mortality. Recently, Beltrán-Sánchez et al. (2008) connected cause-elimination techniques with decomposition methods.

All of these approaches addressed substantively important research questions. Yet their development was motivated by the very specific type of mortality decline envisioned. Consequently, their development was largely done independently of one another; little opportunity arose to deduce

connections. For example, the framework of mortality decline, either hypothetical or retrospective, leads to two seemingly different approaches. Only recently have their equivalence been demonstrated (Beltrán-Sánchez et al., 2008).

In this paper, we provide a unifying framework using functional calculus and demonstrate that previous approaches may be derived from a common formulation. We are now able to concisely study how any conceivable change in age and cause-specific mortality contributes to changes in life expectancy. We apply our method to violent deaths, the leading cause of death among young adults. In doing so, we provide an important tool for policymakers to assess the impact of a potential safety campaign, targeted to particular age groups, on life expectancy.

2 Theoretical Derivations

2.1 Functionals

We use functional differential calculus to link changes in age and cause-specific mortality with changes in life expectancy. Functional differential calculus was first applied to demography by Arthur (1984). A function $f : \mathcal{A} \rightarrow \mathcal{B}$ is a mapping that consists of two sets \mathcal{A} and \mathcal{B} with a rule that assigns to each element $a \in \mathcal{A}$ a specific element of \mathcal{B} , which is denoted as $f(a)$. Similarly, a functional F is a mapping from a vector space, \mathcal{F} , to the field underlying the vector space (usually the real numbers \mathbb{R}), which is denoted as $\mathcal{F}[f]$. Whereas a function is an element-by-element mapping; a functional maps an entire function to an element. For example, we can define the probability of surviving from birth to age a at time t , $p(a, t) = \exp\left(-\int_0^a \mu(s, t) ds\right)$ where $\mu(s, t)$ corresponds to the hazard rate at age s time t , using a functional of the form $p(a, t) = \exp(F[\mu])$ where $F[\mu]$ is defined as

$$F[\mu] = -\int_0^a \mu(s, t) ds. \tag{1}$$

$F[\mu]$ assigns to every function μ a real number corresponding to the negative of the definite integral of that function μ from 0 to a . Notice that μ could be any function of hazard rates, for example, a Gompertz function. Using the functional F , life expectancy at birth at time t , $e(0, t)$, can be written as $\int_0^{\infty} e^{F[\mu]} da$.

2.2 Functional Differentials

The functional differential of $F[f]$, denoted by $\delta F[f; h]$, approximates the change in F when f changes by a function h (Luenberger, 1968). This functional differential is defined as:

$$\delta F[f; h] = \lim_{\alpha \rightarrow 0} \frac{F(f + \alpha h) - F(f)}{\alpha}. \quad (2)$$

In general, when a model explicitly expresses a variable of interest, say r , in terms of functions f_i and parameters x_j for $i = 1, \dots, n$ and $j = 1, \dots, m$ such that $r = F(f_1, f_2, \dots, f_n, x_1, x_2, \dots, x_m)$, then we can write the differential change in r as

$$\delta r = \sum_{i \in I} \delta r[\delta f_i] + \sum_{j \in J} \delta r[\delta x_j]. \quad (3)$$

where I and J represent the set of functions and parameters indices, respectively, which may change (Arthur, 1984). In other words, the differential change in the variable r corresponds to the sum of differential changes in functions and parameters. When the variable of interest r is implicitly expressed in the model, we have an implicit functional model of the form: $0 \equiv F(r, f_1, f_2, \dots, f_n, x_1, x_2, \dots, x_m)$. Thus, we have the following functional differential:

$$0 = \delta F[\delta r] + \sum_{i \in I} \delta F[\delta f_i] + \sum_{j \in J} \delta F[\delta x_j]. \quad (4)$$

For example, suppose our variable of interest is the probability of survival from birth to age a at time t , $p(a, t)$. In this case we have an explicit model linking $p(a, t)$ with $\mu(a, t)$. Our functional F takes a single argument, a function $\mu(a, t)$, as shown in equation 1. Further, suppose we want to find the change on this survivorship when there is a change in $\mu(a, t)$. Using equation 3 and the

chain rule, the functional change in $p(a, t)$ is defined as:

$$\delta p(a, t) = \delta e^{F[\mu]} = \frac{\partial(e^F)}{\partial F} \delta F[\delta\mu] = e^F \left[- \int_0^a \delta\mu(s, t) ds \right] = -p(a, t) \int_0^a \delta\mu(s, t) ds. \quad (5)$$

Note that $\delta\mu(a, t)$ expresses a change in the force of mortality that may occur along both the age *and* time dimensions. Thus, from equation 2, we can define $\delta\mu(a, t)$ as

$$\delta\mu(a, t) = \begin{cases} \lim_{\alpha \rightarrow 0} \frac{\mu(a + \alpha h, t) - \mu(a, t)}{\alpha} \\ \lim_{\alpha \rightarrow 0} \frac{\mu(a, t + \alpha h) - \mu(a, t)}{\alpha}, \end{cases} \quad (6)$$

where the upper case represents a change over the age dimension and the lower case over the time dimension.

2.3 First Derivations

As discussed in Section 2.1, $e(0, t)$ is equal to $\int_0^\infty e^{F[\mu]} da$. Let G be a functional defined as:

$$G[\mu, t] = \int_0^\infty e^{F[\mu]} da - e(0, t) \equiv 0.$$

where $F[\mu]$ is defined in equation 1. Using equation 4, the functional differential of G is given by:

$$0 = \delta G[\delta\mu] + \delta G[\delta t]. \quad (7)$$

The first term of equation 7 is equivalent to:

$$\delta G[\delta\mu] = \int_0^\infty \delta e^{F[\mu]} da = \int_0^\infty \frac{\partial e^{F[\mu]}}{\partial F} \delta F[\delta\mu] da, \quad (8)$$

after applying the chain rule. Following the logic of the previous example (see equation 5) equation 8 reduces to:

$$\begin{aligned}
\delta G[\delta\mu] &= - \int_0^\infty e^F \int_0^a \delta\mu(s, t) ds da \\
&= - \int_0^\infty \int_0^a p(a, t) \delta\mu(s, t) ds da \\
&= - \int_0^\infty \delta\mu(s, t) \int_s^\infty p(a, t) da ds \\
&= - \int_0^\infty \delta\mu(s, t) p(s, t) e(s, t) ds.
\end{aligned} \tag{9}$$

The second term of equation 7 is equivalent to:

$$\delta G[\delta t] = - \frac{\partial e(0, t)}{\partial t} \delta t. \tag{10}$$

Substituting equations 9 and 10 into equation 7 we obtain:

$$0 = - \int_0^\infty \delta\mu(s, t) p(s, t) e(s, t) ds - \frac{\partial e(0, t)}{\partial t} \delta t.$$

Thus,

$$\frac{\partial e(0, t)}{\partial t} = - \int_0^\infty \frac{\delta\mu(s, t)}{\delta t} p(s, t) e(s, t) ds. \tag{11}$$

Finally, dividing equation 11 by $e(0, t)$ we obtain the relative change in life expectancy at birth, $\acute{e}(0, t)$, due to a change in the force of mortality:

$$\acute{e}(0, t) = \frac{- \int_0^\infty \frac{\delta\mu(s, t)}{\delta t} p(s, t) e(s, t) ds}{e(0, t)}. \tag{12}$$

2.4 Multiple Causes of Death

Let μ_1, \dots, μ_N be the set of mutually exclusive and exhaustive causes of death. Replacing μ with its sum $\mu_1 + \dots + \mu_N$ in equation 11, the continuous change in life expectancy at birth due to

changes in cause-specific mortality equals:

$$\frac{\partial e(0, t)}{\partial t} = - \int_0^{\infty} \frac{\sum_{i=1}^N \delta \mu_i(s, t)}{\delta t} p(s, t) e(s, t) ds. \quad (13)$$

Similarly, replacing μ with its sum in equation 12, the relative change in life expectancy at birth due to changes in cause-specific mortality equals:

$$\acute{e}(0, t) = \frac{- \int_0^{\infty} \frac{\sum_{i=1}^N \delta \mu_i(s, t)}{\delta t} p(s, t) e(s, t) ds}{e(0, t)}. \quad (14)$$

2.5 Particular Cases

The expressions $\delta \mu(s, t)/\delta t$ and $\delta \mu_i(s, t)/\delta t$ in equations 11, 12, 13, and 14 represent the changes in all cause and cause-specific force of mortality, respectively, for every unit of time. It is precisely these quantities and their evaluation under different contexts that provide a unifying framework linking changes in life expectancy with changes in forces of mortality. For example, we may be interested in estimating how much of the observed change in life expectancy between two time periods is attributed to changes in a particular cause of death (Arriaga, 1982; Pollard, 1982). Similarly, we may also be interested in evaluating the change in life expectancy under two different mortality scenarios (Keyfitz, 1977). In these cases, the functional differential of $\mu(s, t)$ and $\mu_i(s, t)$ would be evaluated in the context of a discrete change over time or its equivalent—a discrete change between two different scenarios. Additionally, we may be interested in both of the above questions from a continuous time perspective (Vaupel, 1986; Vaupel and Canudas Romo, 2003; Beltrán-Sánchez et al., 2008). Then, the functional differential of $\mu(s, t)$ and $\mu_i(s, t)$ would be evaluated in the context of a continuous time framework. Equations 11, 12, 13, and 14 precisely represent these approaches in a concise and unifying framework. We fully explore these ideas and their relationship with previous work in the next section.

2.5.1 Discrete Case

First, suppose there is a new scenario in which the force of mortality is proportionally reduced at all ages, i.e. $\mu^*(a, t) = (1 + k)\mu(a, t)$ where k is a small negative number (see Keyfitz, 1977). The functional differential of $\mu(a, t)$ is equal to its change between the two scenarios,

$$\delta\mu(a, t) = k\mu(a, t). \quad (15)$$

Additionally, in the discrete case, $\frac{\partial e(0, t)/\partial t}{e(0, t)}$ is equivalent to the difference in life expectancies between the two scenarios divided by the original life expectancy:

$$\frac{[e^*(0, t) - e(0, t)]/\delta t}{e(0, t)} = \frac{\partial e(0, t)/\partial t}{e(0, t)}.$$

Then, substituting equation 15 into 12, equation 12 reduces to

$$\frac{e^*(0, t) - e(0, t)}{e(0, t)} = -\frac{\int_0^{\infty} k\mu(s, t)p(s, t)e(s, t)ds}{e(0, t)} \quad (16)$$

given that in the discrete case we have only one time unit change, $\delta t = 1$. Thus, from equation 16 we derive the well known result of proportional change in life expectancy by Keyfitz (1977, p.413):

$$\begin{aligned} \frac{e^*(0, t)}{e(0, t)} &= 1 - k \frac{\int_0^{\infty} \mu(s, t)p(s, t)e(s, t)ds}{e(0, t)} \\ &= 1 - k \left[\frac{\int_0^{\infty} p(s, t)\ln p(s, t)ds}{e(0, t)} \right] \\ &= 1 - kH. \end{aligned} \quad (17)$$

The equivalence of the first and second lines in the above equation were shown by Vaupel (1986) and Goldman and Lord (1986).

Now suppose there is a new scenario in which the cause-specific force of mortality is proportionally reduced at all ages, i.e. $\mu_i^*(a, t) = (1 + k)\mu_i(a, t)$ for cause i , where k is a small negative number (see Keyfitz, 1977). In this case, the functional differential of $\mu_i(a, t) = k\mu_i(a, t)$. Following the

same approach as in all-cause mortality, one can show that equation 14 reduces to another well known result of proportional change in life expectancy by Keyfitz (1977, p.414):

$$\begin{aligned}
\frac{e^*(0, t)}{e(0, t)} &= 1 - k \frac{\int_0^{\infty} \sum_{i=1}^N \mu_i(s, t) p(s, t) e(s, t) ds}{e(0, t)} \\
&= 1 - k \sum_{i=1}^N \left(- \frac{\int_0^{\infty} p(s, t) \ln p_i(s, t) ds}{e(0, t)} \right) \\
&= 1 - \sum_{i=1}^N k H_i.
\end{aligned} \tag{18}$$

Second, suppose there is an absolute improvement ϕ in the force of mortality $\mu(a, t)$ for some $a \in [x, x + \Delta x]$, i.e. $\mu^*(a, t) = \mu(a, t) + \phi$, where ϕ is a small negative number (see Pollard, 1982). Then, the functional differential of $\mu(a, t)$ is equal to ϕ . That is:

$$\delta\mu(a, t) = \phi = \phi + \mu(a, t) - \mu(a, t) = \mu^*(a, t) - \mu(a, t). \tag{19}$$

Similarly, in this discrete case, $\partial e(0, t)/\partial t$ is equivalent to $\frac{e^*(0, t) - e(0, t)}{\delta t}$, which reduces to $e^*(0, t) - e(0, t)$ given that we have only one time unit change, $\delta t = 1$. Substituting equation 19 into equation 11, equation 11 reduces to the well known result for attributing absolute changes in mortality and the corresponding absolute changes in life expectancy by Pollard (1982):

$$\begin{aligned}
e^*(0, t) - e(0, t) &= - \int_0^{\infty} \phi p(s, t) e(s, t) ds = - \int_0^{\infty} [\mu^*(s, t) - \mu(s, t)] p(s, t) e(s, t) ds \\
&= \int_0^{\infty} [\mu(s, t) - \mu^*(s, t)] p(s, t) e(s, t) ds.
\end{aligned} \tag{20}$$

Now suppose there is an absolute improvement $\frac{\phi}{N}$ for each cause-specific force of mortality such that $\delta\mu_i = \frac{\phi}{N}$. Following the same approach as in all-cause mortality, equation 14 reduces to the

cause-specific mortality result of Pollard (1982):

$$\begin{aligned}
e^*(0, t) - e(0, t) &= - \int_0^\infty \sum_{i=1}^N \delta\mu_i(s, t) p(s, t) e(s, t) ds \\
&= - \int_0^\infty \sum_{i=1}^N \frac{\phi}{N} p(s, t) e(s, t) ds \\
&= - \int_0^\infty \sum_{i=1}^N [\mu_i^*(s, t) - \mu_i(s, t)] p(s, t) e(s, t) ds \\
&= \int_0^\infty \sum_{i=1}^N [\mu_i(s, t) - \mu_i^*(s, t)] p(s, t) e(s, t) ds.
\end{aligned} \tag{21}$$

2.5.2 Continuous Case

First, suppose there is continuous progress against all-cause mortality with respect to time such that the rate of progress in $\mu(a, t)$ is given by $\rho(a, t) = \frac{\partial\mu(a, t)/\partial t}{\mu(a, t)}$ (Vaupel, 1986). Then, the ratio of the functional differential of μ to that of time, $\frac{\delta\mu(a, t)}{\delta t}$, is equivalent to $\frac{-\partial\mu(a, t)}{\partial t}$. Thus, equation 12 reduces to the well known result of Vaupel (1986):

$$\begin{aligned}
\pi(t) = \frac{\partial e(0, t)/\partial t}{e(0, t)} &= - \frac{\int_0^\infty \frac{-\partial\mu(s, t)}{\partial t} p(s, t) e(s, t) ds}{e(0, t)} = \frac{\int_0^\infty \rho(s, t) \mu(s, t) p(s, t) e(s, t) ds}{e(0, t)} \\
&= \int_0^\infty \rho(s, t) \eta(s, t) ds,
\end{aligned} \tag{22}$$

where $\eta(s, t) = \mu(s, t) p(s, t) e(s, t)/e(0, t)$.

Similarly, when there is a continuous progress against cause-specific mortality with respect to time, $\sum_{i=1}^N \frac{\delta\mu_i(a, t)}{\delta t}$ is equivalent to $\sum_{i=1}^N \frac{-\partial\mu_i(a, t)}{\partial t}$. Thus, equation 14 reduces to

$$\begin{aligned}
\frac{\partial e(0, t)}{\partial t} &= \int_0^\infty \frac{\sum_{i=1}^N \partial\mu_i(s, t)}{\partial t} p(s, t) e(s, t) ds \\
&= \sum_{i=1}^N \int_0^\infty \frac{\partial\mu_i(s, t)}{\partial t} p(s, t) e(s, t) ds,
\end{aligned} \tag{23}$$

which is the Vaupel and Canudas Romo (2003) result for decomposing changes in life expectancy by causes of death.

Second, suppose we are interested in the change with respect to time in the gain in life expectancy at birth when one cause of death is eliminated. This question was recently addressed by Beltrán-Sánchez et al. (2008). The years of life gained at birth at time t if cause of death i is eliminated is computed as $D_i(0, t) = e_{-i}(0, t) - e(0, t)$, where $e_{-i}(0, t)$ represents life expectancy at birth at time t when cause of death i is eliminated. Then, the change in $D_i(0, t)$ with respect to time is given by

$$\frac{\partial D_{-i}(0, t)}{\partial t} = \frac{\partial e_{-i}(0, t)}{\partial t} - \frac{\partial e(0, t)}{\partial t}. \quad (24)$$

Both terms on the right hand side of equation 24 are particular cases of equation 11. Thus,

$$\frac{\partial e_{-i}(0, t)}{\partial t} = - \int_0^{\infty} \frac{\delta \mu_{-i}(s, t)}{\delta t} p_{-i}(s, t) e_{-i}(s, t) ds = - \int_0^{\infty} \frac{\partial \mu_{-i}(s, t)}{\partial t} p_{-i}(s, t) e_{-i}(s, t) ds \quad (25)$$

because the ratio of functional differentials, $\frac{\delta \mu_{-i}(s, t)}{\delta t}$, is equal to $\frac{\partial \mu_{-i}(s, t)}{\partial t}$. Similarly,

$$\begin{aligned} \frac{\partial e(0, t)}{\partial t} &= - \int_0^{\infty} \frac{\delta \mu(s, t)}{\delta t} p(s, t) e(s, t) ds = - \int_0^{\infty} \frac{\partial \mu(s, t)}{\partial t} p(s, t) e(s, t) ds \\ &= - \int_0^{\infty} \frac{\partial \mu_i(s, t)}{\partial t} p(s, t) e(s, t) ds - \int_0^{\infty} \frac{\partial \mu_{-i}(s, t)}{\partial t} p(s, t) e(s, t) ds. \end{aligned} \quad (26)$$

Substituting equations 25 and 26 into equation 24 leads to the main result of Beltrán-Sánchez et al. (2008) for linking decomposition approaches and cause-deleted life tables:

$$\begin{aligned} \frac{\partial D_i(0, t)}{\partial t} &= - \int_0^{\infty} \frac{\partial \mu_{-i}(s, t)}{\partial t} p_{-i}(s, t) e_{-i}(s, t) ds + \int_0^{\infty} \frac{\partial \mu_i(s, t)}{\partial t} p(s, t) e(s, t) ds + \int_0^{\infty} \frac{\partial \mu_{-i}(s, t)}{\partial t} p(s, t) e(s, t) ds \\ &= \int_0^{\infty} \frac{\partial p_{-i}(s, t)}{\partial t} ds - \int_0^{\infty} \frac{\partial p_i(s, t)}{\partial t} p_{-i}(s, t) ds - \int_0^{\infty} \frac{\partial p_{-i}(s, t)}{\partial t} p_i(s, t) ds \\ &= \int_0^{\infty} \frac{\partial p_{-i}(s, t)}{\partial t} [1 - p_i(s, t)] ds - \int_0^{\infty} \frac{\partial p_i(s, t)}{\partial t} p_{-i}(s, t) ds. \end{aligned} \quad (27)$$

2.6 Varying Proportional Declines By Age and Cause of Death

Revisiting our main substantive research question, suppose we are interested in the change in life expectancy if there had been declines in particular age-specific motor vehicle, homicide, and suicide

mortality rates over time. Our quantity of interest is life expectancy at birth. Under this scenario, we suppose there are targeted proportional reductions in the force of mortality for specific ages and causes. Let $\mathcal{A}_1, \dots, \mathcal{A}_m$ be the set of targeted age groups for $j = 1, \dots, m$. Let cause one be motor vehicle accident mortality, cause two be homicide, cause three be suicide, and cause four be all other causes of death, so that $\mu(a, t) = \sum_{i=1}^4 \mu_i(a, t)$ for any given age a at time t . Let $k_{i, \mathcal{A}_j} \leq 0$ be the proportional reduction in mortality for cause i and age group \mathcal{A}_j , $j = 1, \dots, m$. As discussed in Section 2.5.1, the functional differential of $\mu_i(a, t)$ is equal to:

$$\delta\mu_i(a, t) = \begin{cases} k_{i, \mathcal{A}_j} \mu_i(a, t) & \text{for } j = 1, \dots, m \\ 0 & \text{otherwise.} \end{cases} \quad (28)$$

In other words, the change in cause-specific mortality after the targeted reduction is proportional to the original cause-specific mortality for ages within the targeted age range and is zero otherwise. Substituting equation 28 into equation 13, the change in life expectancy at birth between the two scenarios is equal to:

$$\begin{aligned} e^*(0, t) - e(0, t) &= - \left[\int_{\mathcal{A}_1} \sum_{i=1}^4 k_{i, \mathcal{A}_1} \mu_i(s, t) p(s, t) e(s, t) ds + \dots + \int_{\mathcal{A}_m} \sum_{i=1}^4 k_{i, \mathcal{A}_m} \mu_i(s, t) p(s, t) e(s, t) ds \right] \\ &= \sum_{j=1}^m \left[\sum_{i=1}^4 (-k_{i, \mathcal{A}_j}) \int_{\mathcal{A}_j} \mu_i(s, t) p(s, t) e(s, t) ds \right]. \end{aligned} \quad (29)$$

The integral in the above equation represents the potential years of life lost (YLL) at time t due to cause i in age group \mathcal{A}_j . A similar quantity, aggregated over cause, was previously noted by Vaupel (1986). The scalar $-k_{i, \mathcal{A}_j}$ represents the recovery of potential years of life lost for cause i age group \mathcal{A}_j . For example, if $-k_{i, \mathcal{A}_j} = 0.80$, we recover 80% of the potential years of life lost at time t due to cause i in age group \mathcal{A}_j . Finally, the product represents the realized years of life gained at time t due to cause i in age group \mathcal{A}_j .

Although the theoretical derivation of the change in life expectancy at birth is given within the continuous-time framework, the data are typically recorded in a discrete form. Let a_{jstart} and a_{jend}

represent the starting and ending ages of the age group \mathcal{A}_j , i.e., $\mathcal{A}_j = [a_{j_{start}}, a_{j_{end}}]$. The width of this age group is equal to $n_j = a_{j_{end}} - a_{j_{start}}$. Then, using life table notation, equation 29 may be reexpressed as:

$$e^*(0, t) - e(0, t) \approx \sum_{j=1}^{m-1} \left[\sum_{i=1}^4 -k_{i, \mathcal{A}_j} \frac{n_j d_{a_{j_{start}}, i}}{l_0} \frac{e_{a_{j_{start}}} + e_{a_{j_{end}}}}{2} \right] \quad (30)$$

where l_0 represents the life table radix; $n_j d_{a_{j_{start}}, i}$ is the number of deaths from cause i in age group \mathcal{A}_j in the life table for cause i ; $e_{a_{j_{start}}}$ is life expectancy at age $a_{j_{start}}$ (the starting age of age group \mathcal{A}_j) and $e_{a_{j_{end}}}$ is life expectancy at age $a_{j_{end}}$ (the ending age of age group \mathcal{A}_j) in the life table for all-cause mortality, respectively. The term $\frac{n_j d_{a_{j_{start}}, i}}{l_0} \frac{e_{a_{j_{start}}} + e_{a_{j_{end}}}}{2}$ represents the estimated potential years of life lost at time t due to cause i in age group \mathcal{A}_j .

3 Applications

3.1 Data

We calculated 1970 and 2005 death counts for the total United States (U.S.) population from the Mortality Detail Files, which contain information on all deaths registered on individual U.S. death certificates transmitted to the National Center for Health Statistics. Deaths were disaggregated by age and sex, as well as the following causes: motor vehicle accidents, homicides, suicides, and all other causes. Hereafter, we refer to motor vehicle accident mortality, homicides, and suicides as violent deaths. Comparable codes for these causes of death were derived from the Centers for Disease Control and Prevention (Centers for Disease Control and Prevention, 2001). We use exposure-to-risk calculated by the Human Mortality Database University of California, Berkeley (USA) and Max Planck Institute for Demographic Research (Germany) (2006). Finally, we combine death counts and exposure-to-risk to calculate mortality rates by age, sex, and cause for 1970 and 2005. The terminal age category begins at age 100.

Considerable progress had been made against violent deaths for most of the ages between 1970

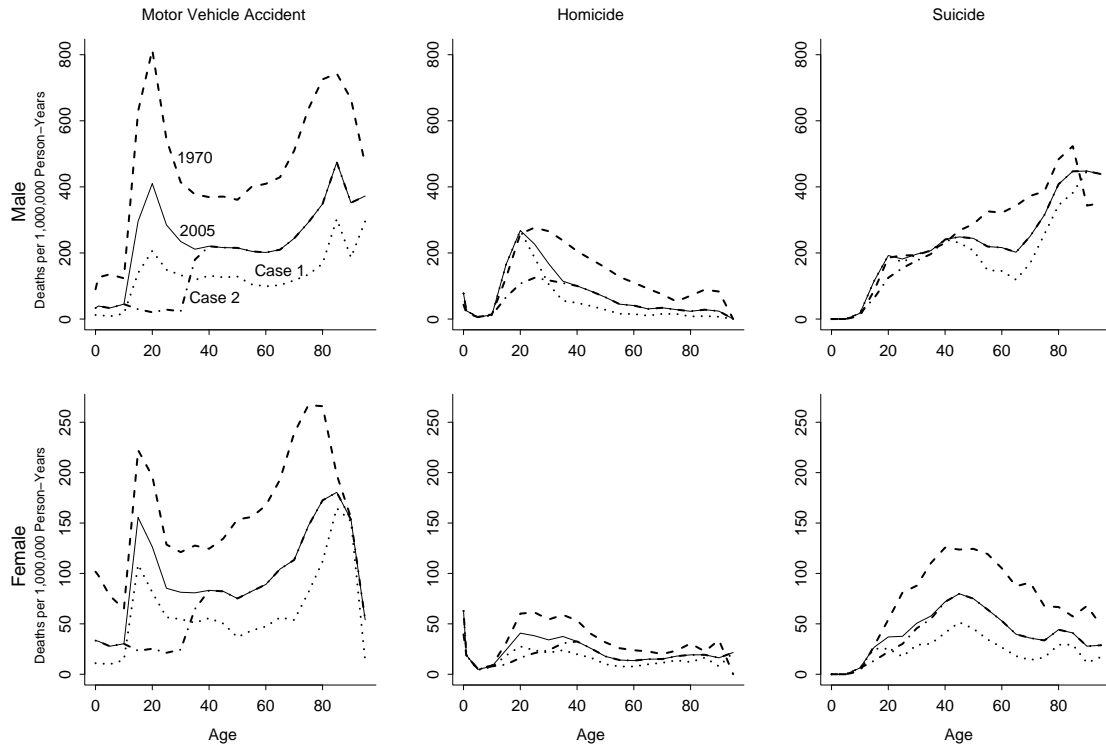
Table 1: Observed Percentage Change in Age- and Sex-Specific Violent Death Mortality Rates For Total U.S. Population Between 1970 and 2005.

Age Group	Male			Female		
	Motor Vehicle Accidents	Homicide	Suicide	Motor Vehicle Accidents	Homicide	Suicide
0-1	-63.1	78.7	0.0	-67.2	58.8	0.0
1-4	-68.7	35.2	0.0	-66.6	1.3	0.0
5-9	-75.4	11.3	1.1	-64.4	-13.3	0.0
10-14	-63.5	-15.1	85.7	-53.3	-8.7	110.9
15-19	-52.9	32.0	33.9	-30.0	-22.6	-5.2
20-24	-49.7	4.0	3.5	-35.7	-32.3	-31.7
25-29	-47.6	-18.0	-5.6	-33.7	-37.8	-53.6
30-34	-43.3	-37.0	1.6	-32.9	-37.3	-42.4
35-39	-44.0	-51.8	-0.4	-36.7	-36.4	-47.6
40-44	-40.3	-51.2	4.4	-33.1	-39.1	-42.8
45-49	-41.6	-53.4	-7.2	-38.9	-36.1	-35.4
50-54	-40.3	-57.6	-15.1	-51.1	-44.0	-39.8
55-59	-48.8	-64.7	-32.7	-47.3	-45.5	-46.2
60-64	-50.8	-62.9	-32.8	-46.8	-43.2	-49.7
65-69	-51.1	-66.5	-41.1	-46.0	-34.0	-54.5
70-74	-52.1	-54.5	-32.7	-52.6	-24.7	-60.5
75-79	-54.0	-46.2	-18.5	-44.9	-23.2	-50.3
80-84	-52.0	-66.7	-15.6	-35.1	-37.0	-33.7
85-89	-36.1	-68.2	-14.6	-8.7	-14.7	-28.5
90-94	-47.1	-71.2	30.4	-1.1	-52.0	-59.0
95-99	-20.5	0.0	24.9	-75.1	NA	-40.2
100+	-67.7	NA	NA	NA	-100.0	NA

Note: For age groups and causes in which no deaths occurred in 1970 and 2005, the change is not applicable (NA). Source: Mortality Detail Files, 1968 to 2005, and Human Mortality Database.

and 2005, as shown in Table 1. For example, improvements in vehicle safety and the introduction of car seats, especially rear-facing car seats, in part led to a more than 60% decline in motor vehicle accident mortality for children under age 10 in this period. Notable exceptions are male and female infanticide and male late adolescent homicide and suicide. For example, infanticides increased by 78% and 59% for males and females between 1970 and 2005, respectively.

Figure 1: Age Profiles of U.S. Male and Female Motor Vehicle Accident, Homicide, and Suicide Mortality Rates: 1970 (dashed), 2005 (solid), Case 1 (dotted), and Case 2 (dotted-dashed).



Source: Mortality Detail Files, 1968-2005 and Human Mortality Database.

3.2 Case 1

Suppose we are interested in the change in life expectancy at birth if the same proportional reductions as historically observed in Table 1 are again applied to 2005 mortality rates. For example, there was a 75.4% decline in male age 5-9 motor vehicle accident mortality between 1970 and 2005; in this new scenario, the 2005 male motor vehicle accident mortality rate for this age group is reduced by another 75.4%. If the mortality rate increased between 1970 and 2005, it remains at the 2005 level in the new scenario. The matrix of k_{i,A_j} values for this scenario is identical to Table 1 except that positive values are replaced with 0. We present the mortality rates of this new scenario in Figure 1, along with historically observed 1970 and 2005 rates. We also calculate period life expectancy in 2005, as well as under this new mortality scenario in Table 2.

Table 2: Change in Male and Female Life Expectancy at Birth: Case 1 and Case 2 Compared to Year 2005.

	Year 2005	Case 1		Case 2	
	e_0 (1)	New Scenario e_0 (2)	Difference (3)=(2)-(1)	New Scenario e_0 (4)	Difference (5)=(4)-(1)
Male	74.91	75.28	0.37	75.36	0.45
Female	80.49	80.66	0.17	80.63	0.14

Note: Change in Male and Female Life Expectancy at Birth: Case 1 and Case 2 Compared to Year 2005. Source: Authors' calculations based on data from Mortality Detail Files and Human Mortality Database.

We estimate the gain in life expectancy at birth under this scenario using equation 30 and present results in Table 3. The estimated potential years of life lost (\widehat{YLL}) by age and cause are shown in columns (2), (5), and (8). The age and cause-specific reductions, which form the $-k_{i,A_j}$ matrix, are shown in columns (1), (4), and (7). The pairwise product that represents the estimated years of life gained under this scenario is shown in columns (3), (6), and (9). Finally, column (10) represents the contribution of each age group to the total gain in life expectancy.

Columns (2), (5), and (8) show that, among men in 2005, violent deaths were responsible for 1.40 years of life lost: 0.62 years from motor vehicle accidents, 0.31 years from homicides, and 0.47 years from suicides. Applying the mortality reductions shown in columns (1), (4), and (7) leads to a gain in life expectancy at birth for men of about 0.30 years from motor vehicle accidents, 0.07 years from homicides, and about 0.04 years from suicides, for a total of about 0.42 years of life gained (column 10). Even when we apply these large declines in age and cause-specific mortality rates we are only able to recover about half, one-fourth and one-tenth of the potential years of life lost from motor vehicle accidents, homicides and suicides, respectively. Our estimate of the years of life gained (0.42 years for males and 0.19 years for females) differs slightly from those calculated in Table 2. The differences are due to the non-independence among causes of death, especially in older adult age groups, as well as the discrete approximation of Equation 29.

Table 3: Estimated Gain in Life Expectancy at Birth Assuming a Proportional Change in Age and Cause-Specific Mortality Rates for Year 2005.

Age Group	Motor Vehicle Accidents			Homicide			Suicide			Total
	$-k_{A_j}$ (1)	\widehat{YLL} (2)	(3)= (1)×(2)	$-k_{A_j}$ (4)	\widehat{YLL} (5)	(6)= (4)×(5)	$-k_{A_j}$ (7)	\widehat{YLL} (8)	(9)= (7)×(8)	(10)= (3)+(6)+(9)
Male										
0-1	0.631	0.002	0.002	-0.000	0.006	-0.000	-0.000	0.000	-0.000	0.002
1-4	0.687	0.011	0.008	-0.000	0.007	-0.000	-0.000	0.000	-0.000	0.008
5-9	0.754	0.011	0.009	-0.000	0.002	-0.000	-0.000	0.000	-0.000	0.009
10-14	0.635	0.014	0.009	0.151	0.004	0.001	-0.000	0.005	-0.000	0.010
15-19	0.529	0.086	0.046	-0.000	0.048	-0.000	-0.000	0.034	-0.000	0.046
20-24	0.497	0.110	0.055	-0.000	0.072	-0.000	-0.000	0.052	-0.000	0.055
25-29	0.476	0.070	0.033	0.180	0.056	0.010	0.056	0.045	0.003	0.046
30-34	0.433	0.052	0.023	0.370	0.037	0.014	-0.000	0.044	-0.000	0.036
35-39	0.440	0.042	0.018	0.518	0.023	0.012	0.004	0.041	0.000	0.030
40-44	0.403	0.039	0.016	0.512	0.018	0.009	-0.000	0.043	-0.000	0.025
45-49	0.416	0.033	0.014	0.534	0.013	0.007	0.072	0.039	0.003	0.024
50-54	0.403	0.029	0.012	0.576	0.009	0.005	0.151	0.033	0.005	0.022
55-59	0.488	0.023	0.011	0.647	0.005	0.003	0.327	0.025	0.008	0.023
60-64	0.508	0.019	0.010	0.629	0.004	0.002	0.328	0.021	0.007	0.019
65-69	0.511	0.016	0.008	0.665	0.002	0.002	0.411	0.016	0.006	0.016
70-74	0.521	0.015	0.008	0.545	0.002	0.001	0.327	0.015	0.005	0.014
75-79	0.540	0.014	0.008	0.462	0.001	0.001	0.185	0.015	0.003	0.011
80-84	0.520	0.013	0.007	0.667	0.001	0.001	0.156	0.015	0.002	0.010
85-89	0.361	0.013	0.005	0.682	0.001	0.001	0.146	0.012	0.002	0.007
90-94	0.471	0.007	0.003	0.712	0.001	0.000	-0.000	0.009	-0.000	0.004
95-99	0.205	0.006	0.001	-0.000	0.000	-0.000	-0.000	0.007	-0.000	0.001
Total	-	0.627	0.303	-	0.312	0.068	-	0.470	0.044	0.415
Female										
0-1	0.672	0.003	0.002	-0.000	0.005	-0.000	-0.000	0.000	-0.000	0.002
1-4	0.666	0.010	0.007	-0.000	0.006	-0.000	-0.000	0.000	-0.000	0.007
5-9	0.644	0.010	0.007	0.133	0.002	0.000	-0.000	0.000	-0.000	0.007
10-14	0.533	0.010	0.006	0.087	0.003	0.000	-0.000	0.002	-0.000	0.006
15-19	0.300	0.050	0.015	0.226	0.008	0.002	0.052	0.009	0.000	0.017
20-24	0.357	0.037	0.013	0.323	0.012	0.004	0.317	0.011	0.003	0.021
25-29	0.337	0.023	0.008	0.378	0.010	0.004	0.536	0.010	0.005	0.017
30-34	0.329	0.020	0.007	0.373	0.008	0.003	0.424	0.012	0.005	0.015
35-39	0.367	0.018	0.007	0.364	0.008	0.003	0.476	0.013	0.006	0.016
40-44	0.331	0.016	0.005	0.391	0.006	0.003	0.428	0.014	0.006	0.014
45-49	0.389	0.014	0.006	0.361	0.004	0.002	0.354	0.014	0.005	0.012
50-54	0.511	0.011	0.006	0.440	0.003	0.001	0.398	0.011	0.005	0.012
55-59	0.473	0.011	0.005	0.455	0.002	0.001	0.462	0.008	0.004	0.010
60-64	0.468	0.010	0.005	0.432	0.001	0.001	0.497	0.006	0.003	0.008
65-69	0.460	0.009	0.004	0.340	0.001	0.000	0.545	0.004	0.002	0.007
70-74	0.526	0.008	0.004	0.247	0.001	0.000	0.605	0.003	0.002	0.006
75-79	0.449	0.008	0.004	0.232	0.001	0.000	0.503	0.002	0.001	0.005
80-84	0.351	0.007	0.003	0.370	0.001	0.000	0.337	0.002	0.001	0.004
85-89	0.087	0.006	0.000	0.147	0.001	0.000	0.285	0.001	0.000	0.001
90-94	0.011	0.003	0.000	0.520	0.000	0.000	0.590	0.001	0.000	0.001
95-99	0.751	0.001	0.001	-0.000	0.000	-0.000	0.402	0.000	0.000	0.001
Total	-	0.287	0.112	-	0.085	0.024	-	0.123	0.049	0.186

Note: Columns 1, 4, and 7 correspond to proportionate reductions in age and cause-specific mortality. Columns 2, 5, and 8 correspond to the estimated maximum potential years of life gained for complete reduction, \widehat{YLL} . Columns 3, 6, and 9 correspond to the estimated realized years of life gained under Case 1 reductions by sex, age, and cause. Column 10 corresponds to the total estimated realized years of life by age and sex. Source: authors' calculations.

Of course, if the outcome of interest is simply the total gain in life expectancy at birth when violent death mortality is reduced by a supposed amount, then the calculation of a life table under this new scenario would suffice. Yet from a policy perspective, it is equally important to know for which ages and for which specific causes a change in mortality rates would produce the greatest gain in life expectancy at birth.

3.3 Case 2

A closer look at table 3 shows that the greatest loss of potential life years occurs between ages 15 and 34 for most of the violent deaths. Thus, we can imagine a new scenario in which we focus on reducing mortality rates at these particular ages. We show this case as a dashed line in Figure 1. Such declines might reflect the result of an aggressive and targeted public health and safety campaign. For example, we assume a decline of 95% in motor vehicle accidents, 60% in homicide, and 35% in suicide among 20-24 year old men (Table 4). We then estimate the highest gain in life expectancy at birth would occur for the 20-24 year old age group.

We achieve a similar gain in life expectancy through these targeted age and cause-specific reductions as in case 1. For males, targeted motor vehicle accident, homicide, and suicide reductions contribute about 0.30, 0.11, and 0.05 years of life expectancy, respectively (Table 4). Similarly for females, the corresponding gains are 0.11, 0.02, and 0.01 years of life expectancy. This leads to a total gain in life expectancy at birth of 0.45 years and 0.14 years for males and females, respectively. Compared to Case 1, our estimated years of life gained (0.45 years for males and 0.14 years for females) are virtually identical to those calculated in Table 2. In this case, reductions were targeted at ages in which the assumption of independence among causes of death is far more plausible.

In both cases, we utilize our common formulation to link changes in life expectancy with changes in the forces of mortality. For each case, we simply evaluate its particular change in cause-specific forces of mortality for every unit of time, $\delta\mu_i(s,t)/\delta t$. One could certainly envision other cases of

Table 4: Estimated Gain in Life Expectancy at Birth Assuming a Targeted Proportional Change in Age and Cause-Specific Mortality Rates for Year 2005.

Age Group	Motor Vehicle Accidents			Homicide			Suicide			Total
	$-k_{\mathcal{A}_j}$ (1)	\widehat{YLL} (2)	(3)= (1)×(2)	$-k_{\mathcal{A}_j}$ (4)	\widehat{YLL} (5)	(6)= (4)×(5)	$-k_{\mathcal{A}_j}$ (7)	\widehat{YLL} (8)	(9)= (7)×(8)	(10)= (3)+(6)+(9)
Male										
15-19	0.900	0.086	0.078	0.600	0.048	0.029	0.450	0.034	0.015	0.122
20-24	0.950	0.110	0.104	0.600	0.072	0.043	0.350	0.052	0.018	0.166
25-29	0.900	0.070	0.063	0.450	0.056	0.025	0.150	0.045	0.007	0.094
30-34	0.900	0.052	0.047	0.300	0.037	0.011	0.100	0.044	0.004	0.062
35-39	0.150	0.042	0.006	0.050	0.023	0.001	0.050	0.041	0.002	0.010
Total		0.627	0.298		0.312	0.109		0.470	0.046	0.454
Female										
15-19	0.850	0.050	0.042	0.600	0.008	0.005	0.500	0.009	0.004	0.051
20-24	0.800	0.037	0.030	0.600	0.012	0.007	0.400	0.011	0.004	0.041
25-29	0.750	0.023	0.017	0.450	0.010	0.005	0.200	0.010	0.002	0.024
30-34	0.700	0.020	0.014	0.300	0.008	0.003	0.100	0.012	0.001	0.018
35-39	0.200	0.018	0.004	0.180	0.008	0.001	0.050	0.013	0.001	0.006
Total		0.287	0.107		0.085	0.020		0.123	0.013	0.140

Note: Columns 1, 4, and 7 correspond to proportionate reductions in age and cause-specific mortality. Columns 2, 5, and 8 correspond to the estimated maximum potential years of life gained for complete reduction, \widehat{YLL} . Columns 3, 6, and 9 correspond to the estimated realized years of life gained under Case 2 reductions by sex, age, and cause. Column 10 corresponds to the total estimated realized years of life by age and sex. Source: authors' calculations.

hypothesized mortality reduction, as well. Once $\delta\mu_i(s,t)/\delta t$ is evaluated in this new context, the same methodology developed in Section 2 applies.

4 Concluding Remarks

In this paper, we develop two important results. First, we provide a unifying framework for assessing the change in life expectancy given any conceivable change in age and cause-specific mortality. Two conceptualizations of mortality change are possible: a counterfactual assessment of a hypothetical change or a retrospective assessment of an observed change. Our framework, developed in Sections 2.3 and 2.4, shows how these conceptualizations can be easily implemented to assess changes in life expectancy with respect to time, both for all-cause and multiple cause of

death. In doing so, we thus connect previous demographic research into a sound and parsimonious formulation.

Second, when we apply our methodology to a particular case of targeted age and cause-specific mortality reductions, we obtain an especially useful byproduct: the maximum potential years of life lost, specific to each age and cause, given current mortality. For policymakers, this quantity provides an estimate of the theoretical maximum years of life that could be potentially recovered if all deaths in this age and cause could be averted. In a particular case, only a proportion of deaths would be averted; then only this proportion of years of life would be recovered. For example, the estimated maximum years of life lost due to male 20-24 year old motor vehicle accidents is 0.11 years in 2005. If we could decline mortality in this cause and age group by 65%, we would recover 0.07 years of life expectancy at birth.

An important limitation in this area of demographic research is the assumption of independence among causes of death. Whereas the assumption may hold for violent deaths among young adults, it is far more tenuous for other causes of death among older adults. For age groups 15 to 34, which we study in Section 3.3, violent deaths are the leading cause of death. The risk of death from other causes, notably leading causes among the elderly (cardiovascular disease, cancer, and stroke) is considerably smaller. For example, if a young adult's motor vehicle accident death could have been averted, the decedent's risk of death from other causes would likely change very little. On the other hand, if an older adult's diabetes death could have been averted, this decedent's risk of death from other causes will likely change because of diabetes-related comorbidities. Further work is needed to consider the non-independence among causes of death.

Our methodological and substantive results have immediate implications for policymakers. They often require an assessment of possible gains in life expectancy that could result from a public health campaign aimed at reducing mortality for specific ages and causes of death. We

provide a theoretical framework and simple tools¹ that inform these important policy decisions.

¹The **R** functions of the proposed method are available from the authors upon request.

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